

NatPar: Natural Parametric Modeling*

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Abstract

We develop *natural parametric* (NatPar) insurance as the natural next step from natural-catastrophe (NatCat) modelling: the same hazard–exposure–vulnerability–finance machinery, with a parametric index made contractual in place of indemnity loss adjustment. Our aim is practical—to establish a *standard approach*, inspired by how NatCat models and the catastrophe-insurance industry already operate, rather than to propose another optimal-contract criterion. This vantage point delivers two payoffs. First, it fixes *how the reporting is formulated*: NatPar contracts are reported in the native NatCat language—annual average loss, EP/AEP/OEP curves, return-period levels, one-year tail metrics—and complemented with two-sided basis-exceedance diagnostics (BEP[±] and portfolio basis AEP/OEP) that we elevate to the same central status the EP curve holds for losses: BEP[±] is the canonical distributional view of basis risk, not a supplementary number. We package these into a minimal reporting template that makes designs comparable, auditable, and usable by underwriting, capital, and supervisory functions rather than living only in abstract contract-design space. Second, the same standard lets us see *how the tail is reallocated* between insuree and insurer. As a pricing tool we pose trigger pricing as a single loss-minimisation over the two-sided basis, shaping the premium through both a weight and a *curvature* on shortfall versus overpayment; AAL-neutral (“fair”) pricing is the symmetric-linear corner of this problem, a special case rather than a separate paradigm.

A frost case study with three simulated regions sharing one identical published damage function illustrates the framework and yields its central result, which is about *time rather than average*. Holding a contract AAL-neutral, the bounded payout cannot follow the unbounded exposure-severity tail, so equalising the mean forces over- and under-payment to separate across return periods: the insuree gains at short and moderate horizons while the insurer sheds the deep tail past a crossover of several decades. This relocation sharpens as exposure growth and climate change fatten the loss tail, but reverses once hazards exhibit *tail dependence*—when regions reach their extremes jointly, bounded payouts stack and the insurer reabsorbs the deep tail. The incidence of catastrophe risk under NatPar is thus the resultant of two opposing climate forces, neither visible to a correlation-based portfolio model, giving a precise account of why bounded parametric structures have grown in popularity and where their governance attention belongs.

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1 Introduction

Over the last decade, climate pressure alongside rapid economic growth has increased both exposure and hazard intensity across many regions. In several markets, climate-exposed lines have moved toward outcomes increasingly described—informally or explicitly—as uninsurable: coverage is withdrawn, terms are tightened, or prices rise to levels that are politically and socially untenable. At the same time, protection gaps for climate perils remain large and, in some settings, continue to widen.

From the perspective of insurers and reinsurers, the binding challenge is often not only higher mean losses, but reduced tractability of the loss distribution, especially in the tail. Hazard distributions drift, exposures evolve quickly, and extremes may cluster in space and time; these dynamics inflate uncertainty precisely where solvency frameworks and risk appetite concentrate. Long-tailed indemnity liabilities become harder to reserve, harder to explain, and harder to support under one-year capital constraints and target returns. In this sense, a line drifts toward uninsurability when markets cease to clear because one or more constraints becomes binding: affordability (premiums exceed willingness or ability to pay), capital (tail requirements render the line uneconomic), and/or model uncertainty (pricing and reserving assumptions are no longer defensible to management, regulators, or capacity providers). These constraints reinforce each other: uncertainty in exposure and vulnerability increases capital charges and risk loadings, pushing premiums higher and accelerating capacity withdrawal.

Against this background, parametric structures have expanded not merely as customer-facing innovations, but as a supply-side reconfiguration of NatCat practice. Under climate-driven uninsurability and capital pressure, NatCat teams increasingly redeploy the hazard–exposure–vulnerability–finance stack to produce index-linked liabilities that are auditable, bounded by contract, and more capital-tractable. We refer to this pipeline as Natural Parametric (NatPar) modelling. Conceptually, NatPar preserves the NatCat architecture but changes what becomes contractual: hazard modelling defines measurable indices and triggers; exposure and vulnerability are re-tasked into a basis-risk engine that quantifies the mismatch between realised indemnity loss and parametric payout; and the finance block is adapted to short-tailed liabilities with minimal development risk. NatPar therefore exchanges a portion of exposure-driven indemnity variability for a transparent, index-verified payout, reducing reserve uncertainty and improving capital tractability at the cost of explicit basis risk.

Design-first versus pipeline-first. It is useful to contrast our objective—to set standards for NatPar reporting and regulation—with the dominant framing in much of the academic literature

and industrial development.

- **Academic view: design-first.** A large literature treats parametric insurance as a contract-design problem: specify an index (or index vector) X and choose a payout function $I(X)$ to optimise a welfare or risk objective under a premium constraint. This perspective is mathematically flexible and emphasises basis risk as the mismatch between realised loss and index-based payout, making it well suited for studying optimality, robustness, behavioural frictions, and uptake.
- **Industry view: pipeline-first.** In practice, however, many parametric products are developed inside the NatCat ecosystem. Underwriting, reinsurance/ILS structuring, accumulation control, and solvency reporting rely on a modular workflow (hazard, exposure, vulnerability, financial terms) and a standard set of portfolio outputs (AAL and EP/AEP/OEP curves, return-period levels, and one-year tail metrics). From this viewpoint, a parametric contract is not an arbitrary $I(X)$: it must be verifiable, auditable, and comparable within the same reporting language used for indemnity portfolios and capital.

This creates a practical gap between design-first theory and pipeline-first implementation—a gap that becomes acute when products must be justified to boards, supervisors, and capacity providers.

This paper takes the view that NatPar is the *natural next step from NatCat*: not a new object grafted onto catastrophe modelling, but the same hazard–exposure–vulnerability–finance machinery with a different question asked of it—what should become contractual. Our aim is correspondingly practical. Rather than proposing yet another optimal-contract criterion, we set out to **establish a standard approach**—one deliberately inspired by how NatCat models and the catastrophe-insurance industry already work—so that parametric contracts can be built, reported, and governed with the tools practitioners already use. The contribution is therefore methodological and infrastructural rather than a single theorem: a way of working that is comparable to indemnity practice, auditable by supervisors, and usable by underwriting and capital functions. Adopting this vantage point delivers two concrete payoffs, around which the paper is organised: it fixes *how the reporting is formulated*, and it lets us see *how the tail is reallocated* between the parties.

Both payoffs rest on a single methodological departure. Almost the entire NatCat apparatus carries over untouched—the hazard, exposure, and vulnerability blocks, the AAL and EP/AEP/OEP outputs, the capital metrics. The one thing that is genuinely new when an index becomes contractual is the mismatch between the indemnity loss and the parametric payout, the *basis risk*, and our central claim is about how to look at it: as a *distribution*, reported with the same exceedance-and-return-period machinery already used for losses, rather than collapsed into a single correlation or hedge-effectiveness number. This distributional reading of basis risk is what makes the reporting standard auditable and what makes the tail-reallocation result visible; the two payoffs below are its two consequences.

The first payoff is **how the reporting is formulated**: a standard built on the basis-risk distribution. Because NatPar lives inside the NatCat stack, its parametric contracts are constructed and reported with the conventional portfolio outputs that the industry already produces—AAL, EP/AEP/OEP curves, return-period levels, and one-year tail metrics—so they are directly comparable to indemnity programmes rather than living in a separate analytical world. The one genuinely new object is basis risk, and the practical choice we make is to report it not as a single summary number but as a *distribution*, through two-sided basis-exceedance curves ($\text{BEP}^\pm$) for shortfall and overpayment, with their portfolio (BAEP/BOEP) and return-period counterparts. The emphasis matters: we give $\text{BEP}^\pm$ the same central status that the EP curve has on the loss side. It is the canonical view of basis risk—the object the report is built around and the one against which

contract shapes are compared—not a supplementary tail diagnostic bolted on beside the loss EP curve. Wherever the existing workflow reaches for an EP curve, the NatPar workflow reaches for the matching BEP[±]. We compute these under an AAL-neutral calibration so that the short-fall/overpayment structure reflects payout shape rather than a difference in mean loss, and we package the whole into a minimal reporting template—a concrete, repeatable checklist that makes NatPar designs comparable, auditable, and amenable to supervision.

The second payoff is that the same standard lets us see **how the tail is reallocated**. Holding contracts AAL-neutral isolates the effect of payout shape on the tail: contracts with the same mean loss can have materially different tails, so AAL-neutrality does not imply capital-neutrality under VaR-style solvency measures. More strikingly, the bounded parametric payout cannot follow the unbounded exposure-severity tail of the indemnity loss, so equalising the mean *relocates* mismatch across return periods rather than removing it. This is a concrete, reportable statement about the *allocation of risk between insuree and insurer* over time—the insuree tends to gain at short horizons while the insurer sheds the deep tail at long ones—and, because it is expressed in the same return-period language as the rest of the report, it is exactly the kind of thing a board or regulator can read off directly. We show how diversification and, crucially, tail dependence among hazards move that allocation.

Underpinning both payoffs is a pricing tool we develop in passing but use throughout: **trigger pricing posed as loss-minimisation**. The premium is the solution of minimising the expected disutility of the two-sided basis, shaped by both a weight and a *curvature* on shortfall versus overpayment (more generally, any convex loss). Asymmetric curvature relocates the premium along the shortfall/overpayment trade-off; the symmetric-linear corner of this problem coincides with AAL-neutral (“fair”) pricing, which we therefore treat as a special case rather than a separate paradigm. We illustrate all of this in a single case study with three simulated regions sharing one identical damage function, applying a consistent reporting template throughout.

The remainder of the paper is organised as follows. Section 2 reviews the contract-design and multi-hazard index-insurance literatures and contrasts them with the pipeline and reporting language used in catastrophe practice. Section 3 summarises the NatCat hazard–exposure–vulnerability–finance stack and the standard portfolio outputs used for underwriting and capital. Section 4 then formalises the NatPar mapping and introduces the minimal reporting template. Section 5 poses trigger pricing as a loss-minimisation over the basis. Sections 6, 7 and 8 implement the framework in a frost case study with three simulated regions sharing one identical damage function: Section 6 develops the analytic representations, Section 7 reports the empirical outputs for both individual risks and a three-region portfolio, and Section 8 interprets the results. Section 10 discusses extensions, regulatory implications, and practical limitations, and Section 11 concludes.

2 Literature review

Much of the academic literature treats parametric insurance as a generic contract-design problem, whereas the industry has largely evolved parametric programmes as pipeline-compatible overlays on NatCat machinery.

A standard academic entry point models insurance design as a choice of a payout schedule subject to pricing constraints, often derived from expected-utility considerations and classic optimal insurance results (e.g., generalized deductibles) (Arrow, 1974; Raviv, 1979). Index (parametric) insurance adopts a related contract-design perspective, but replaces loss adjustment by a payout determined by an observable index (or index vector) X , typically written as $I(X)$. Modern treatments formalize the choice of $I(\cdot)$ under premium constraints and study welfare, risk, and

implementability of index contracts (Zhang et al., 2019).

Across this literature, basis risk—the mismatch between realized indemnity-style loss and the index-based payout—is consistently identified as the central friction that can dominate performance and suppress demand. Two influential directions are (i) multi-scale indices to reduce mismatch across spatial/temporal aggregation and (ii) empirical measurement of index quality and basis-risk outcomes in the field (Elabed et al., 2013; Jensen et al., 2016). Complementary work emphasises implementation and uptake constraints, including regulatory challenges and behavioural frictions such as complexity aversion, which can reduce uptake even when expected values are favorable (Sonsino & Mandelbaum, 2001; Sonsino et al., 2002).

A conceptually close strand to our motivation is the review by Benso et al. (2023) on weather index insurance for multi-hazard resilience and food security. Beyond synthesizing the literature, they propose a practical three-module framework for index insurance design: hazard identification, vulnerability assessment, and financial methods and risk pricing. This decomposition is a useful bridge between a broad (and sometimes fragmented) academic literature and the practical steps required to build indices and payout rules.

A key message in the multi-hazard setting is that hazard representation and combination are modelling choices, not merely data choices. Benso et al. (2023) emphasise that multi-hazard products should not default to independence assumptions and highlight interaction taxonomies (independent, synergistic, cascading) that can materially affect loss modelling and premium adequacy (Benso et al., 2023; Gill & Malamud, 2014). Related work reviews quantitative methodologies for multi-hazard interrelationships and provides a structured view of how hazard interactions can be represented and tested (Tilloy et al., 2019). In climate-risk contexts, compound-event research provides additional conceptual and statistical tools for thinking about multivariate extremes and interacting drivers (Zscheischler et al., 2020).

In industry practice, catastrophe risk is commonly represented via a modular supply chain: hazard (event sets/footprints), exposure (assets at risk and where), vulnerability (damage/loss functions), and financial terms (deductibles, limits, aggregates and layers). This architecture is central to how portfolios are priced (e.g., AAL) and how tails are communicated (e.g., EP/AEP/OEP curves and return-period losses), and it is described in practitioner-oriented treatments of catastrophe risk modelling (Mitchell-Wallace et al., 2017).

Recent climate physical risk assessment (PRA) and climate change risk assessment (CCRA) work builds naturally on the same decomposition, but adds scenario conditioning, spatial-temporal resolution constraints, and climate-model uncertainty. In particular, portfolio-scale CCRA requires separating changes in hazard from changes in exposure, because their interaction can magnify or attenuate losses (Bourget et al., 2024). Integrating climate scenarios into NatCat-style workflows typically requires explicit choices around downscaling and bias correction, which can materially influence hazard projections and therefore pricing and capital metrics (Chen et al., 2021; Maraun & Widmann, 2018; Martynov et al., 2013; Martel et al., 2020). At the same time, climate disclosure and supervisory stress testing have accelerated demand for portfolio-level methodologies that are transparent and auditable (FSB, 2017; BoE, 2019; OSFI, 2023).

The contract-design literature explains parametric insurance in the abstract language of $I(X)$ and basis risk, and multi-hazard reviews provide useful design taxonomies (Zhang et al., 2019; Benso et al., 2023). However, these literatures do not typically track how parametric products have evolved operationally inside the insurance ecosystem: namely, as contractual overlays on the catastrophe-modelling pipeline that already governs underwriting, portfolio aggregation, reinsurance/ILS structuring, and solvency reporting.

This paper therefore adopts a pipeline-first notion of parametric insurance. We define NatPar programmes as contracts whose triggers, payouts, and validation procedures are explicitly designed

to be compatible with the NatCat supply chain and its portfolio outputs. In this view, basis risk is not merely a single summary statistic but the measurable interface created when (exposure $\times$ vulnerability) is replaced by a low-dimensional, index-verifiable hazard proxy. This is also where model governance lives: NatPar design should be readable in the same reporting language as NatCat (AAL, EP/AEP/OEP, tail metrics), with explicit basis distributions and basis exceedance curves as diagnostics.

3 NatCat machinery in practitioner terms

This section summarises the catastrophe-modelling workflow that underpins NatCat underwriting and portfolio reporting. We focus on the modelling blocks and the portfolio outputs that are used in practice, because NatPar is defined in this paper as a pipeline-compatible overlay on the same machinery.

3.1 The four modeling blocks: hazard, exposure, vulnerability, financial terms

A standard NatCat model can be represented as a modular pipeline:

$$\text{Hazard} \rightarrow \text{Exposure} \rightarrow \text{Vulnerability} \rightarrow \text{Financial terms}.$$

The purpose of this decomposition is operational: each block can be updated, validated, and governed with its own data sources and model risk controls, while still producing consistent portfolio outputs.

Hazard. The hazard block specifies an event set (or stochastic process) for the peril, including frequency, intensity, and spatial footprints where relevant. In climate settings, the hazard block may also be conditioned on scenarios or nonstationary assumptions.

Exposure. The exposure block specifies what is at risk and where: locations, sums insured/values, relevant asset attributes, and aggregation rules. Exposure is often the most rapidly changing component in practice, and it is also a major driver of accumulation risk.

Vulnerability. The vulnerability block maps hazard intensity (and modifiers) to damage ratios or loss distributions. It captures construction/asset sensitivity, secondary uncertainty, and model error.

Financial terms. Financial terms transform ground-up loss into contractual loss: deductibles, limits, layers, aggregates, reinstatements, and other features that define the insurer or reinsurer liability.

3.2 Portfolio loss

Let Y_r denote the contractual loss after applying financial terms for the set of covered risk $r \in \mathcal{R}$. Here $\mathcal{R}$ is a set of risks, that can be for example a deterministic set of regions (as we use in this paper), or even a random set of events (such as $r \in \mathcal{R} \equiv \{1 \leq r \leq N_t\}$ for a Poisson process N_t).

Portfolio-level reporting commonly distinguishes:

$$\text{Occurrence loss: } M := \max_r Y_r, \quad \text{Aggregate loss: } S := \sum_r Y_r.$$

These are the random quantities that generate standard NatCat outputs such as OEP and AEP curves. NatCat portfolio reporting distinguishes between exceedance probabilities for annual occurrence loss (OEP) and annual aggregate loss (AEP). Using the annual variables the one-year curves are defined as

$$\text{OEP}(x) := \mathbb{P}(M > x), \quad \text{AEP}(x) := \mathbb{P}(S > x), \quad x \geq 0.$$

In general, these curves differ because a year can contain multiple loss-causing events: OEP captures the tail of the single largest event, while AEP captures the tail of the sum of events. They coincide only in special cases (e.g. when there is at most one loss-causing event per year in the model, or when financial terms collapse event losses into a single outcome).

Return periods. A common presentation uses return periods. For any liability X , define the T -year return level

$$x_T(X) := \inf\{x : \mathbb{P}(X > x) \leq 1/T\}.$$

Return-period curves $T \mapsto x_T(\cdot)$ provide an interpretable mapping from tail probability to severity. For a return period T (years), the corresponding OEP return-period loss level x_T^{OEP} satisfies $\text{OEP}(x_T^{\text{OEP}}) = 1/T$, and similarly for AEP.

4 NatPar framework

NatPar programmes are defined here as parametric structures anchored in the catastrophe-modelling workflow and evaluated in the same portfolio language as NatCat (AAL, EP/AEP/OEP curves, return periods, tail metrics). The central additional object is basis risk, which we treat as an auditable interface rather than an afterthought. Figure 1 summarises the mapping: NatPar retains the four-block NatCat pipeline but re-tasks what each block contributes contractually.

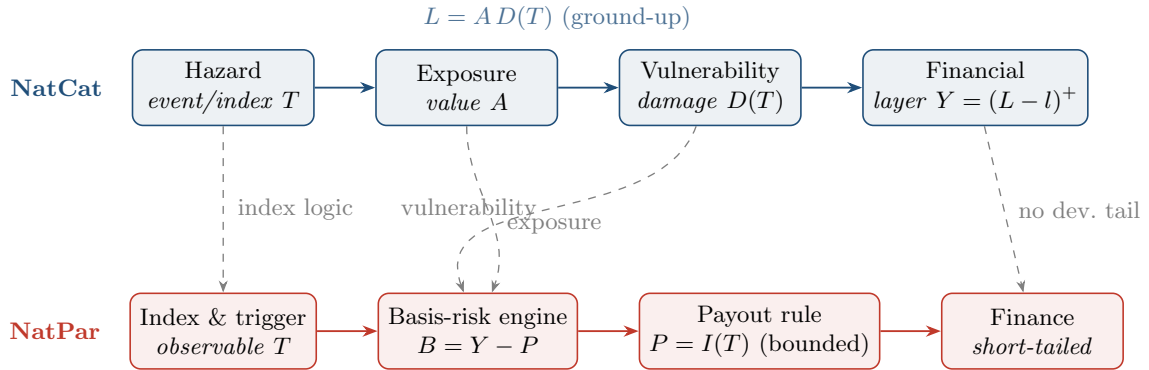


Figure 1: From NatCat to NatPar: the same four-block pipeline, re-tasks. The hazard becomes the contractual index; the exposure and vulnerability blocks are re-tasked into a basis-risk engine that measures the mismatch $B = Y - P$ between indemnity loss and parametric payout; and the finance block becomes short-tailed with limited development risk. The architecture is unchanged — what changes is which object becomes contractual.

4.1 NatPar payout classes and their implications for tails and basis

A practical advantage of a pipeline-first view is that NatPar designs can be organised into a small number of payout classes, each with predictable implications for tail metrics and basis risk. This section provides a compact taxonomy used later to interpret the case study.

(i) Continuous bounded payouts (graded contracts). Contracts of the form $P = I(X)$ with I continuous and bounded (e.g. $P = q(D(X) - d)^+$ with $D \in [0, 1]$) mechanically cap the insurer’s liability and often reduce high-quantile tail metrics relative to exposure-scaled indemnity losses, even under AAL-neutral calibration. Their basis risk tends to be diffuse: frequent moderate mismatch can occur because payouts do not scale with realised exposure, but severe basis shortfalls can be controlled by the cap and by shaping $I(\cdot)$.

(ii) Piecewise-linear / tiered payouts. Piecewise-linear or tiered payouts provide additional flexibility while remaining transparent and auditable. They can be used to approximate layer-like behaviour (e.g. increasing payout rate as hazard severity increases) and to enforce constraints such as $\text{VaR}_{0.995}(P) \leq c$ while still targeting basis shortfall objectives. In a governance setting, the breakpoints of $I(\cdot)$ become explicit design parameters that can be stress-tested and versioned.

(iii) Trigger-driven events. A trigger-driven event (or simply binary) $P = q\mathbf{1}\{X \in \text{event}\}$, for a particular event set and index X for a payout q , is maximally simple and highly verifiable, and they often align with operational objectives such as rapid cashflow upon a threshold event. However, their loss distribution is discrete, which can have unintuitive implications: for a trigger probability $1 - \alpha$ (i.e., $\mathbb{P}\{X \in \text{event}\} = 1 - \alpha$), tail metrics can “jump” at confidence levels above α (e.g. $\text{VaR}_{0.995}$ may equal q when $\alpha = 0.99$). As a result, AAL-neutrality does not guarantee capital advantages, and these type of binary designs can concentrate basis mismatch into rare but large shortfalls or overpayments depending on how q is calibrated.

(iv) Layer-like parametric designs. Between continuous and binary extremes lie parametric designs that emulate insurance layers (e.g. step functions with multiple thresholds, or capped linear segments). These are often the most directly comparable to indemnity layer structures and can be tuned to match both AAL and a tail constraint, while keeping the trigger logic auditable.

4.2 NatCat to NatPar through AAL-neutral calibration

Let Y denote the NatCat contractual loss (for a policy, region, or portfolio segment), and let T denote an index (typically a hazard proxy or near-hazard proxy) that is observable and verifiable. A NatPar design specifies a payout rule $P = I(T)$ together with contract terms such as caps, layers, and trigger thresholds. A natural baseline calibration target is AAL-neutrality:

$$\mathbb{E}[P] = \mathbb{E}[Y].$$

This constraint ensures that differences in EP curves and tail metrics primarily reflect distributional shape rather than mean loss. In practice, this can be implemented at different granularities (policy-level, peril-region segment, or portfolio bucket), and it can be supplemented by loadings for expenses and profit. The key point is that AAL-neutrality is a comparability device, not a full design criterion.

Remark 1 (AAL-neutrality relocates tail risk across time, not on average). *Imposing $\mathbb{E}[P] = \mathbb{E}[Y]$ does not make insurer and insuree indifferent; it only equalises the average transfer. Because P is bounded while Y inherits the unbounded exposure-severity tail of AD , holding the mean fixed forces the over- and under-payment mass to redistribute across return periods. The bounded over-payment side concentrates in frequent, mild years, while the unspanned shortfall side migrates into rare, severe years. AAL-neutrality is therefore best read not as a fairness statement but as the device that makes this temporal relocation visible: with the mean held constant, the basis return-period curves of Section 7 isolate when each party is ahead. We return to the economic reading of this relocation, and its link to climate-driven growth in catastrophe risk, in Section 9 after the numerical results.*

4.3 Basis risk as an auditable interface

This is the paper’s major departure. Everything else—the hazard, exposure, and vulnerability blocks, the AAL and EP/AEP/OEP outputs, the one-year capital metrics—is inherited essentially unchanged from NatCat practice. What genuinely changes when an index becomes contractual is that a new object appears, the mismatch between the indemnity loss and the parametric payout, and the central methodological claim of this paper is about *how that object should be looked at*: not as a single summary number (a correlation, an average shortfall, a hedge-effectiveness ratio), but as a *distribution* in its own right, reported with the same exceedance-and-return-period machinery already used for losses. Treating basis risk distributionally is what turns it from an afterthought into a governable, auditable interface, and it is what makes the downstream adjustments to reporting, underwriting, and portfolio management (developed below and in Section 9) possible.

The fundamental trade-off is that P is transparent and bounded but cannot replicate Y point-wise; the resulting mismatch is basis risk. Therefore, define basis risk as

$$B := Y - P.$$

4.3.1 Individual risk basis

Two useful classes of metric to view a basis risk are:

- **Point measures.** Measures that represent the basis risk through a single scalar such as $\mathbb{P}(B > 0)$, $\mathbb{E}[B^+]$, $\mathbb{E}[(-B)^+]$, $\text{Corr}(Y, P)$, etc.
 - $\mathbb{P}(B > 0)$ distinguishes whether the design tends to miss loss states (high shortfall frequency) or to pay in non-loss states (high overpayment frequency).
 - The severities $\mathbb{E}[B^+]$, $\mathbb{E}[(-B)^+]$ are the relevant averages conditional on sign: they quantify the expected size of liquidity missing when loss occurs (shortfall) and the expected size of liquidity excess when it is not needed (overpayment).
 - The indemnity-parametric correlation $\text{Corr}(Y, P)$ indicates how the statistical mismatch between the real payoff and the parametric product can be measured.
- **The central object: the basis exceedance distribution.** Point summaries are useful shorthand, but each collapses the mismatch to a single scalar and can hide whether mismatch is driven by frequent small shortfalls or rare severe ones. The central view of basis risk in this framework is therefore not a scalar but a *distribution*, reported through its exceedance curves exactly as losses are reported through the EP curve:

$$\text{BEP}^+(x) := \mathbb{P}(B > x), \quad \text{BEP}^-(x) := \mathbb{P}(-B > x).$$

We give these the status of a primary reporting object. *The basis exceedance curve $\text{BEP}^\pm$ is to basis risk what the EP curve is to loss: the canonical distributional view, not a supplementary diagnostic.* Just as no catastrophe report would summarise a loss distribution by its mean alone and omit the EP curve, a NatPar report should not summarise basis risk by a correlation or an average shortfall and omit $\text{BEP}^\pm$. Every place the workflow currently reaches for an EP curve, it should now reach for the matching $\text{BEP}^\pm$; the point measures above are read off the curve, not the other way round.

The two views are genuine counterparts. The loss EP curve $\mathbb{P}(Y > x)$ reports the distribution of what the buyer must pay; the basis curves $\text{BEP}^\pm$ report the distribution of what the contract fails to transfer, signed by direction. The shortfall curve $\text{BEP}^+(x) = \mathbb{P}(B > x)$ answers: how often does the indemnity benchmark exceed the parametric payout by more than x ? The overpayment curve $\text{BEP}^-(x) = \mathbb{P}(-B > x)$ answers: how often does the parametric payout exceed the indemnity benchmark by more than x ? Reading $\text{BEP}^\pm$ as the primary object is what separates frequency of mismatch (the level near $x = 0$), severity of mismatch (how slowly the curve decays for large x), and direction (shortfall versus overpayment), and it is what lets two candidate contract shapes be compared even when their AAL is matched—precisely the comparison a single hedge-effectiveness number cannot support.

Return-period interpretation (basis return periods). Just as EP curves are often communicated via return periods, basis EP curves can be summarised by basis return-period levels. For a return period T (years), define the shortfall-basis return-period level b_T^+ by $\text{BEP}^+(b_T^+) = \mathbb{P}(B > b_T^+) = 1/T$, and similarly define the overpayment-basis return-period level b_T^- by $\text{BEP}^-(b_T^-) = \mathbb{P}(-B > b_T^-) = 1/T$. Reporting (b_T^+, b_T^-) for a small set of T values (e.g. $T \in \{10, 20, 50\}$) yields a compact and interpretable summary of basis-tail behaviour that can sit next to the standard loss return-period table.

4.3.2 Portfolio basis: aggregate and occurrence

Let $B_r = Y_r - P_r$ denote the basis for the risk $r \in \mathcal{R}$. In a multi-region portfolio, basis reporting must distinguish between (i) the aggregate mismatch over all regions in the model year, and (ii) the occurrence (worst-region) mismatch that can dominate operational and regulatory concerns. Given a portfolio, define the aggregate basis as

$$S^B = \sum_{r \in \mathcal{R}} B_r = S^Y - S^P.$$

This quantity is natural when the objective is balance-sheet impact and annual aggregate liquidity, and it aligns with AEP-style reporting for liabilities. For basis regulation and suitability, we additionally introduce an occurrence basis that captures the worst regional mismatch within the year:

$$M^B = \max_{r \in \mathcal{R}} B_r.$$

The occurrence basis is the appropriate object when basis risk is interpreted as an operational shortfall/overpayment exposure that can be driven by a single region, even if the portfolio aggregate is moderate. This parallels the NatCat distinction between AEP and OEP, and it is particularly relevant for supervisory review because it measures basis accumulation: the most adverse mismatch that the index can generate in a given year. The associated tail measures are

$$\text{BAEP}^\pm(x) := \mathbb{P}(\pm S^B > x), \quad \text{BOEP}^\pm(x) := \mathbb{P}(\pm M^B > x),$$

with basis return levels defined analogously by $\text{BAEP}^\pm(b_T^\pm) = 1/T$ and $\text{BOEP}^\pm(b_T^\pm) = 1/T$.

4.4 Minimal reporting template

For each NatPar candidate design (within a region/segment/etc), report:

1. **Mean comparability:** AAL as the calibration target.
2. **Tail comparability:** EP/AEP/OEP, return-period levels.
3. **Basis interface (central object):** the basis exceedance distribution $\text{BEP}^\pm$ (and portfolio $\text{BAEP}^\pm/\text{BOEP}^\pm$) with return-period levels, reported with the same prominence as the loss EP curve; point diagnostics are read off it.
4. **Capital metrics:** $\text{VaR}_{0.995}$ (and TVaR where feasible).
5. **Pricing:** the trigger price from the loss-minimisation of Section 5, with the chosen penalty weight and curvature reported alongside it.

As we will see in Section 7 we implement this reporting template in a frost case study with three simulated regions sharing one identical damage function. We compare a continuous bounded NatPar design to an indemnity benchmark and then compare an extreme indemnity layer to a binary design, illustrating how payout shape can dominate tail capital even under AAL-neutral calibration.

4.5 How the workflow adjusts to basis-risk distribution analysis

Because the distributional view of basis risk is the one genuine departure from NatCat practice, it is worth being explicit about how the three operational functions that consume catastrophe output—reporting, underwriting, and portfolio management—change once basis risk is reported as a distribution rather than as a scalar. In each case the existing workflow is retained; what is added is a basis-distribution analogue of a step the function already performs.

Reporting. The loss report gains a basis report of equal standing. The organising principle is a one-to-one correspondence: every loss-side object acquires a basis-side counterpart reported with the same prominence. The loss EP curve is paired with the two-sided $\text{BEP}^\pm$ curves, the loss return-period table with the basis return-period levels (b_T^+, b_T^-) , and—at portfolio level—the AEP/OEP curves with the $\text{BAEP}^\pm$ and $\text{BOEP}^\pm$ curves. The $\text{BEP}^\pm$ curve is not an appendix to the loss EP curve but its peer: it is the canonical view of the residual risk, and the scalar diagnostics (correlation, average shortfall, hedge effectiveness) are summaries read off it rather than substitutes for it. The reporting unit is unchanged (region, segment, treaty), and AAL-neutral calibration is the comparability convention that lets two candidate designs be placed side by side, since it strips out mean-level differences and leaves only the shape of the mismatch. The practical effect is that a basis report is no longer a single hedge-effectiveness number that can be gamed by averaging; it exposes whether residual risk is frequent-and-small or rare-and-severe, and in which direction (shortfall or overpayment), in the same language the loss report already uses.

Underwriting. Underwriting acceptance criteria, today expressed as thresholds on loss metrics, gain basis-distribution thresholds. Rather than accepting a design because a correlation or hedge-effectiveness ratio clears a cutoff, the underwriter sets limits on the basis tail directly: a maximum admissible shortfall return level b_T^+ at a chosen horizon, a cap on the overpayment return level b_T^- (which is a cost to the writer, not the client), and a sign/severity profile that the contract must respect. Trigger and scale are then chosen by the loss-minimisation of Section 5 subject to

those basis-tail limits, so the curvature of the pricing penalty becomes an explicit underwriting lever: tightening the admissible shortfall tail is the same operation as raising the curvature on the shortfall side. This also disciplines product complexity, because an additional trigger or index is justified only when it produces a measurable reduction in the basis tail net of the transparency and operational robustness it costs.

Portfolio management. At the book level the manager already monitors loss accumulation through AEP and OEP; the basis-distribution analogue is to monitor *basis* accumulation through S^B and M^B . The aggregate basis S^B governs annual balance-sheet and liquidity impact, while the occurrence basis M^B —the worst single-region mismatch in the year—is the object that drives suitability and conduct exposure, because a portfolio whose aggregate basis nets out can still leave one region badly under-covered. Crucially, as developed in Section 9, the basis tail of a book is not the sum of its parts: it depends on how the underlying hazards co-move, and specifically on their *tail dependence* rather than their correlation. Portfolio construction therefore acquires a basis-tail dimension—diversifying across weakly tail-dependent hazards genuinely thins the aggregate basis tail, whereas adding exposures whose extremes co-occur does not—and the manager’s stress tests are best specified as joint-extreme (tail-dependence) scenarios on M^B rather than as correlation matrices on losses.

5 Pricing the trigger by minimising a loss of mismatch

The AAL-neutral calibration of Section 4 fixes the scale of a payout once its shape and trigger are chosen. In this section we promote the trigger to a *decision object* and pose pricing as a single optimisation: choose the contract parameters to minimise the expected disutility of the two-sided mismatch between the parametric payout and the loss it hedges. The shape of that disutility is controlled by the analyst through *both* a weight on each side of the basis *and* a curvature; one may also replace the whole penalty by a general loss function. A particular boundary case of this problem—when the penalty is symmetric and linear—coincides with minimising the mean basis and hence with AAL-neutrality; we record that coincidence as a remark rather than as a separate pricing paradigm.

5.1 Model and notation

Let X denote the parametric index (here the damage proxy D derived from the hazard index T), let $L(X)$ be the true loss the buyer wants to hedge (the NatCat damage loss), and let $P(X; \theta)$ be the parametric payout with contract parameters θ . The basis is $B = P - L$, with under-pay (shortfall) $B^- = (L - P)^+$ and over-pay $B^+ = (P - L)^+$; $\text{AAL} = \mathbb{E}[L]$. In the optimisation the loss curve is fixed exogenously: its anchors are set from index quantiles and its curvature is given, so L is ground truth, not a decision variable.

For the linear-ramp / exceedance contract the payout is

$$P(x; s, e) = N \cdot \text{clip}\left(\frac{x-s}{e-s}, 0, 1\right),$$

i.e. zero below strike s , ramping to the full notional N at exit e . The decision variables are the two triggers $\theta = (s, e)$. The binary payout uses a single trigger, $P = N \cdot \mathbf{1}\{x \geq s\}$, hence one unknown. In the frost case study of Section 7 the same logic is carried by the continuous design $P^c = q^c(D - d)^+$ and the binary design $P^b = q^b \mathbf{1}\{D > d\}$, whose decision variables are $\theta = (d, q)$ and $\theta = (d)$ respectively (with q pinned by AAL-neutrality in the single-trigger case).

5.2 A general loss-minimisation objective

Let ρ be a disutility of mismatch built from the two nonnegative sides of the basis, the shortfall $B^- = (L - P)^+$ and the over-pay $B^+ = (P - L)^+$. We price by

$$\min_{\theta} \rho(L, P(\theta)) := \lambda \mathbb{E}[\phi_-((L - P)^+)] + (1 - \lambda) \mathbb{E}[\phi_+((P - L)^+)], \quad (1)$$

where θ are the contract parameters (the triggers and scale), $\lambda \in [0, 1]$ weights shortfall against over-pay, and $\phi_-, \phi_+ : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are increasing penalties applied to each side. The analyst shapes the price through two distinct levers:

- **Weight** (λ): a linear tilt that makes shortfall more or less costly than over-pay without changing how marginal cost grows with the size of the miss.
- **Curvature** ($\phi_{\pm}$): how fast the penalty accelerates in the size of the miss. A convex ϕ penalises large misses disproportionately, so it reshapes the *tail* of the basis, not merely its average direction.

The leading parametric family is the power penalty $\phi_{\pm}(z) = z^{p_{\pm}}$ with exponents $p_-, p_+ \geq 1$, which nests the symmetric L_1 case $(\lambda, p_-, p_+) = (\frac{1}{2}, 1, 1)$, the shortfall-tilted L_1 case $(\lambda > \frac{1}{2}, 1, 1)$, and asymmetric-curvature cases such as $(p_-, p_+) = (2, 1)$ or $(3, 1)$ that punish under-payment ever more steeply. Nothing in the formulation requires a power penalty: $\phi_{\pm}$ may be any increasing loss—for instance a Huber penalty (quadratic near zero, linear in the tail), an exponential disutility $\phi_-(z) = e^{\kappa z} - 1$ that targets extreme shortfall, or a log-barrier enforcing a hard cap. Equation (1) should therefore be read as a general loss-minimisation, with the power family used only as a transparent default.

In practice we add a soft budget term $\mu |\mathbb{E}[P] - \text{AAL}|$ to keep the premium economically anchored unless the curvature deliberately pulls it away; μ is a Lagrange-style weight, not a separate pricing mode. The problem is solved by a grid over the index quantiles followed by a Nelder–Mead refinement, with expectations evaluated on a Monte-Carlo–simulated index under common random numbers.

Remark 2 (The symmetric-linear case is AAL-neutral pricing). *For any real a , $a^+ - (-a)^+ = a$, so $\mathbb{E}[(P - L)^+] - \mathbb{E}[(L - P)^+] = \mathbb{E}[P] - \text{AAL}$. When the penalty is symmetric and linear $(\lambda, p_-, p_+) = (\frac{1}{2}, 1, 1)$, the gradient of ρ in the scale direction vanishes precisely where $\mathbb{E}[(P - L)^+] = \mathbb{E}[(L - P)^+]$, i.e. where $\mathbb{E}[P] = \text{AAL}$. In that boundary case minimising the mismatch loss is AAL-neutral (“fair”) pricing: equal expected shortfall and over-pay, zero mean basis, and pure-premium scale are one and the same condition. This is why “fair” is not a competing paradigm but the symmetric-linear corner of (1); any asymmetry in weight or curvature moves the price off that corner in a controlled, reportable way. The identity is robust to a payout cap, since $a^+ - (-a)^+ = a$ holds pointwise for the capped P as well.*

5.3 Determinacy: parameters, conditions, and well-posedness

The loss-minimisation (1) has as many free parameters as the payout provides: two for the linear ramp (s, e) (equivalently (d, q) for the continuous frost design), one for the binary trigger. How many *exact* averaging conditions a price can satisfy is bounded by that count. The symmetric-linear corner imposes one condition ($\mathbb{E}[P] = \text{AAL}$, Remark 2); a two-parameter contract therefore retains one degree of freedom, which any asymmetry in weight or curvature spends on shaping the basis:

$$\min_{(s, e)} \lambda \mathbb{E}[\phi_-((L - P)^+)] + (1 - \lambda) \mathbb{E}[\phi_+((P - L)^+)] \quad (\text{soft budget } \mu |\mathbb{E}[P] - \text{AAL}|). \quad (2)$$

Generically this has an isolated (locally unique) solution. The single-trigger binary contract has no spare degree of freedom: the budget alone pins it, so curvature and weight cannot move its price—they can only be reported as the residual basis they leave behind, which is why in the empirical results curvature reshapes the continuous price but not the binary one. The problem becomes over-determined only if one demands more *independent* exact conditions than parameters: with two triggers a second exact condition (e.g. matching loss $\text{TVaR}_{0.95}$) uses the last degree of freedom, and a third leaves no exact solution. The design rule is that exactly-satisfiable conditions must not exceed free parameters; each added parameter (a free cap, a multi-layer attachment, an extra trigger) buys one more, which is also what lets a richer penalty curvature express itself.

Two well-posedness points complete the picture. First, existence: $\mathbb{E}[P(\theta)]$ is continuous and monotone in the triggers and ranges in $[0, N \cdot \mathbb{P}(\text{trigger})]$, so an AAL-neutral solution exists iff AAL lies in that range; if even the most generous admissible payout has $\mathbb{E}[P] < \text{AAL}$ (an under-capitalised or capped contract) the budget is infeasible, and the soft penalty pushing to the boundary is detected and flagged as a signal to re-parameterise—raise the notional or add a layer—rather than sit silently on a boundary. Second, the objective is non-convex (a clip composed with the penalty), so multiple local minima are possible under strong curvature; we therefore solve by a grid followed by Nelder–Mead refinement and report local solutions. Finally, the objective prices at the physical AAL (pure premium); a risk load attaches as $\text{premium} = \text{AAL} + \text{load}$ without disturbing the trigger geometry, since it shifts the budget target rather than the shape of P —equivalently, a convex penalty curvature already encodes a risk attitude endogenously.

6 Two design blocks and one reporting language for individual risk

This section operationalises the NatCat-to-NatPar mapping in a worked frost example on real data and reports outputs in the same portfolio language used in catastrophe practice for an individual risk. The point is not to build the most realistic frost model, but to make the workflow explicit: starting from a NatCat loss specification, constructing a NatPar payout that is legible to underwriting and governance, calibrating it to a transparent target, then constructing the reporting framework and finally comparing the two using the same diagnostics. To better focus on the reporting and regulatory framework, we assume there is one season per region, so that within a region $\text{AEP} = \text{OEP} = \text{EP}$. In the numerical assessment we will use a portfolio in addition to the individual risk.

6.1 Common notations

We index regions by $r \in \{\text{R1}, \text{R2}, \text{R3}\}$. In the frost case study each season produces at most one relevant frost outcome per region. Hence, within a region, annual aggregate and annual occurrence losses coincide, and the usual catastrophe reporting objects (AEP/OEP) reduce to a single exceedance curve. This “one-loss-per-year” structure is deliberate: it isolates the effect of contract design and basis risk without confounding from within-year event counts.

State variables and loss components. For each region r :

- T_r denotes the (seasonal) trigger, e.g., minimum temperature as hazard proxy/index.
- A_r denotes seasonal exposure, e.g., monetary value at risk.

- $D(\tau) \in [0, 1]$ denotes a non-increasing hazard-driven damage fraction (vulnerability curve). We assume $D(\tau) > 0 \Leftrightarrow \tau < \tau_t$, for a given τ_t , and $D(\tau) < 1 \Leftrightarrow \tau > \tau_c$. The left inverse of D is denoted by D^- .
- L_r denotes ground-up loss. In this paper we consider the multiplicative form $L_r = A_r D_r$. We use the notation $D_r = D(T_r)$ throughout.

Let us introduce the following exposure functions: the CDF $F_A(x) := \mathbb{P}(A \leq x)$, and the call functional $C_A(x) := \mathbb{E}[(A - x)^+]$.

NatCat vs. NatPar payoffs. We write contractual (NatCat-type) indemnity loss as a transformation of ground-up loss, $Y_r := \psi(L_r)$, where ψ captures financial terms (e.g. deductible/stop-loss layer). The parametric (NatPar-type) payout depends only on the hazard proxy, $P_r := I(T_r)$, where $I(\cdot)$ is the payout function (continuous, piecewise, or binary).

Calibration convention. Within each region and design block, we calibrate NatPar by an AAL-neutral principle, $\mathbb{E}[P_r] = \mathbb{E}[Y_r]$, so differences in EP curves, tail metrics, and basis risk reflect distributional shape and index mismatch rather than differences in mean payout. This is a comparability device, not a claim of optimality. In Section 7 we additionally report trigger prices from the loss-minimisation of Section 5 under a panel of penalty shapes.

Ground-up loss. The seasonal ground-up loss is multiplicative: $L_r := A_r D(T_r) = A_r D_r$. This is the canonical NatCat construction: hazard produces a state (T), vulnerability maps hazard to a fractional impact (D), and exposure scales impact into monetary loss ($A \times D$). In this form, uncertainty in A loads directly into the loss distribution whenever $D_r > 0$.

NatCat indemnity cover with deductible. We benchmark against a simple deductible (stop-loss) indemnity form with region-specific deductible $l_r \geq 0$:

$$Y_r := (L_r - l_r)^+ = \max(A_r D_r - l_r, 0).$$

Because the frost case study produces at most one relevant frost loss per season in each region, Y_r is simultaneously the event loss, the annual occurrence loss, and the annual aggregate loss for that region.

6.2 NatCat: analytic EP and AAL representations

Fix a region r and a threshold $x \geq 0$. The exceedance probability of indemnity loss is

$$\text{EP}_r^{\text{Cat}}(x) = \text{EP}_r(x) := \mathbb{P}(Y_r > x) = \mathbb{E} \left[\bar{F}_{A_r} \left(\frac{l_r + x}{D_r} \right) \right], \quad (3)$$

where $\bar{F}_{A_r}$ is the exposure survival in region r . This form makes explicit the supply-side sensitivity: tail exceedance is controlled by the probability that exposure exceeds a hazard-dependent threshold proportional to $(l_r + x)/D_r$. Similarly, the average annual loss (AAL) of the deductible indemnity admits the conditional form

$$\text{AAL}_r^{\text{Cat}} = \mathbb{E} \left[D_r C_{A_r} \left(\frac{l_r}{D_r} \right) \right]. \quad (4)$$

Equations (3)–(4) show that, in the ground-up NatCat benchmark, exposure uncertainty enters the liability distribution through F_{A_r} . This is the structural mechanism that NatPar will later

modify: once payout depends only on T_r (and is AAL-calibrated), exposure uncertainty drops out of the insurer's liability distribution, but it reappears as a measurable residual mismatch in the basis variable $B_r = Y_r - P_r$.

6.3 NatPar: analytic EP and AAL representations

The key structural change relative to the NatCat benchmark is that the payout depends only on the observable hazard proxy T_r (through the damage function), not on exposure A_r . The ground-up payouts are

$$L_r^c = (D_r - d_r)^+, \quad L_r^b = \mathbf{1}\{T_r < \tau_r\} = \mathbf{1}\{D_r > d_r\},$$

where $D(\cdot)$ is the same vulnerability curve used in the NatCat benchmark and $d_r = D(\tau_r)$, with τ_r a region-specific trigger. This payoff is (i) fully index-driven, (ii) bounded by construction, and (iii) interpretable.

6.3.1 Continuous parametric payout

Define a continuous payout NatPar as

$$P_r^c := q_r^c L_r^c = q_r^c (D_r - d_r)^+. \quad (5)$$

To compare distributional shape rather than mean level, we choose q_r^c such that NatPar is AAL-neutral to the NatCat benchmark indemnity Y_r :

$$\text{AAL}_r := \mathbb{E}[P_r^c] = \mathbb{E}[Y_r]. \quad (6)$$

Substituting (5) into (6) yields the explicit calibration

$$q_r^c = \frac{\mathbb{E}[(A_r D_r - l_r)^+]}{\mathbb{E}[(D_r - d_r)^+]} = \frac{\mathbb{E}\left[D_r C_A\left(\frac{l_r}{D_r}\right)\right]}{C_{D_r}(d_r)}. \quad (7)$$

Because P_r^c depends only on T_r , its EP curve has a simple hazard-only representation: for any threshold $x \geq 0$,

$$\text{EP}_r^c(x) := \mathbb{P}(P_r^c > x) = \bar{F}_{D_r}\left(\frac{x}{q_r^c} + d_r\right). \quad (8)$$

6.3.2 Binary payout

The NatPar binary payout is

$$P_r^b = q_r^b L_r^b = q_r^b \mathbf{1}\{T_r < \tau_r\}, \quad (9)$$

with AAL-neutral calibration $\mathbb{E}[P_r^b] = \mathbb{E}[Y_r]$. Substituting (9) into the AAL-neutrality condition gives

$$q_r^b = \frac{\mathbb{E}[(A_r D_r - l_r)^+]}{\mathbb{P}\{D_r > d_r\}} = \frac{\mathbb{E}\left[D_r C_A\left(\frac{l_r}{D_r}\right)\right]}{\bar{F}_{D_r}(d_r)}. \quad (10)$$

The exceedance follows as

$$\text{EP}_r^b(x) = \begin{cases} \bar{F}_{D_r}(d_r), & x \leq q_r^b, \\ 0, & x > q_r^b. \end{cases}$$

6.4 Basis risk diagnosis

Let $B_r^{\{\cdot\}} := Y_r - P_r^{\{\cdot\}}$ for $\cdot \in \{c, b\}$.

6.4.1 Point diagnosis

It is not difficult to show that

$$\mathbb{E}[B_r^{\{\cdot\},+}] = \mathbb{E}\left[D_r C_A\left(\frac{l_r + P_r^{\{\cdot\}}}{D_r}\right)\right].$$

Given AAL-neutrality, $\mathbb{E}[B_r^{\{\cdot\}}] = 0$, which implies $\mathbb{E}[B_r^{\{\cdot\},+}] = \mathbb{E}[B_r^{\{\cdot\},-}]$. In addition,

$$\text{Cov}(Y_r, P_r^{\{\cdot\}}) = \mathbb{E}\left[P_r^{\{\cdot\}} D_r C_A\left(\frac{l_r}{D_r}\right)\right] - (\text{AAL}_r^{\{\cdot\}})^2.$$

Continuous NatPar. With P_r^c :

$$\begin{aligned}\mathbb{E}[B_r^{c,+}] &= \mathbb{E}\left[D_r C_{A_r}\left(\frac{l_r}{D_r}\right) \mathbf{1}\{D_r \leq d_r\}\right] + \mathbb{E}\left[D_r C_{A_r}\left(\frac{l_r}{D_r} + q_r^c\left(1 - \frac{d_r}{D_r}\right)\right) \mathbf{1}\{D_r > d_r\}\right], \\ \text{Var}(P_r^c) &= (q_r^c)^2 \left(\mathbb{E}\left[D_r^2 \left(1 - \frac{d_r}{D_r}\right)_+^2\right] - \mathbb{E}\left[D_r \left(1 - \frac{d_r}{D_r}\right)_+\right]^2 \right), \\ \text{Cov}(Y_r, P_r^c) &= q_r^c \mathbb{E}\left[D_r^2 \left(1 - \frac{d_r}{D_r}\right)_+^2 C_A\left(\frac{l_r}{D_r}\right)\right] - (\text{AAL}_r)^2.\end{aligned}$$

Binary NatPar. With P_r^b :

$$\begin{aligned}\mathbb{E}[B_r^{b,+}] &= \mathbb{E}\left[D_r C_{A_r}\left(\frac{l_r}{D_r}\right) \mathbf{1}\{D_r \leq d_r\}\right] + \mathbb{E}\left[D_r C_{A_r}\left(\frac{l_r + q_r^b}{D_r}\right) \mathbf{1}\{D_r > d_r\}\right], \\ \text{Var}(P_r^b) &= (q_r^b)^2 \mathbb{P}(D_r > d_r)(1 - \mathbb{P}(D_r > d_r)), \\ \text{Cov}(Y, P^b) &= q_r^b \left(\mathbb{E}\left[D_r C_{A_r}\left(\frac{l_r}{D_r}\right)\right] - \text{AAL}_r \mathbb{P}(D_r > d_r) \right).\end{aligned}$$

6.4.2 Basis exceedance curve

We treat basis risk as a distributional object via the exceedance-basis curves $\text{BEP}_r^{\{\cdot\},+}(x) := \mathbb{P}(B_r^{\{\cdot\}} > x)$ and $\text{BEP}_r^{\{\cdot\},-}(x) := \mathbb{P}(-B_r^{\{\cdot\}} > x)$ for $x \geq 0$. Here $B_r^{\{\cdot\}} > 0$ is shortfall and $-B_r^{\{\cdot\}} > 0$ is overpayment. Under AAL-neutrality, $\mathbb{E}[B_r^{\{\cdot\}}] = 0$, but this does not control tails. Fixing $x \geq 0$ and conditioning on T_r :

Shortfall exceedance. If $D_r = 0$, then $Y_r = P_r^c = 0$ and $B_r^c = 0$. If $D_r > 0$, $B_r^{\{\cdot\}} > x \Leftrightarrow A_r > (l_r + x + P_r^{\{\cdot\}})/D_r$, hence

$$\text{BEP}_r^{\{\cdot\},+}(x) = \mathbb{E}\left[\bar{F}_{A_r}\left(\frac{l_r + x + P_r^{\{\cdot\}}}{D_r}\right)\right]. \quad (11)$$

Overpayment exceedance. Combining the cases $A_r D_r - l_r > 0$ and $A_r D_r - l_r \leq 0$,

$$\text{BEP}_r^{\{\cdot\},-}(x) = \mathbb{E}\left[F_{A_r}\left(\frac{l_r - x + P_r^{\{\cdot\}}}{D_r}\right) \mathbf{1}\{P_r^{\{\cdot\}} > x\}\right]. \quad (12)$$

6.5 Continuous NatPar basis analysis

Under the baseline independence assumptions:

$$\text{BEP}_r^{c,+}(x) = \mathbb{E}\left[\bar{F}_{A_r}\left(\frac{l_r+x}{D_r} + q_r^c\left(1 - \frac{d_r}{D_r}\right)\right) \mathbf{1}\{D_r > d_r\}\right] + \mathbb{E}\left[\bar{F}_{A_r}\left(\frac{l_r+x}{D_r}\right) \mathbf{1}\{D_r \leq d_r\}\right], \quad (13)$$

$$\text{BEP}_r^{c,-}(x) = \mathbb{E}\left[F_{A_r}\left(\frac{l_r-x}{D_r} + q_r^c\left(1 - \frac{d_r}{D_r}\right)\right) \mathbf{1}\{D_r q_r^c(1 - d_r/D_r) > x\}\right]. \quad (14)$$

6.6 Binary NatPar basis analysis

$$\text{BEP}_r^{b,+}(x) = \mathbb{E}\left[\bar{F}_{A_r}\left(\frac{l_r+x+q_r^b}{D_r}\right) \mathbf{1}\{D_r > d_r\}\right] + \mathbb{E}\left[\bar{F}_{A_r}\left(\frac{l_r+x}{D_r}\right) \mathbf{1}\{D_r \leq d_r\}\right], \quad (15)$$

$$\text{BEP}_r^{b,-}(x) = \begin{cases} \mathbb{E}\left[F_{A_r}\left(\frac{l_r-x+q_r^b}{D_r}\right) \mathbf{1}\{D_r > d_r\}\right], & q_r^b > x, \\ 0, & q_r^b \leq x. \end{cases} \quad (16)$$

7 Reporting the frost risk example

We illustrate the framework on three regions that share a single, identical frost damage function—the convex citrus curve of [Assa et al. \(2017\)](#) with the published parameters—and differ only in their night-minimum temperature law. Because real gridded temperature series for arbitrary locations do not, in general, fall within the narrow citrus damaging band on which the published curve is calibrated, we drive the three regions with *simulated* night-minimum temperatures, positioned so that the same fixed damage function is exercised with graded severity. The three regions are ordered by frost exposure—R1 most exposed, R2 intermediate, R3 least—mirroring the way the three months (January, December, February) of the published San Joaquin study differ in frost risk. The objective of this subsection is twofold: (i) to make the NatCat hazard–exposure–vulnerability loss construction explicit, and (ii) to express key distributional objects (EP curves and AAL) in a conditional form that carries over to basis-risk analysis.

7.1 Hazard–exposure–vulnerability and NatCat indemnity

The frost loss model we use builds directly on the indemnity-based frost-pricing framework of [Assa et al. \(2017\)](#). That paper deliberately prices frost as an *indemnity* contract on the loss—avoiding an index precisely because an index introduces basis risk—using distortion premiums (VaR, CVaR, Wang). Here we take its loss model as the ground-up NatCat loss and then ask the complementary, supply-side question: what happens when one *does* make a bounded index contractual, and reports the resulting basis risk in the NatCat language. The present case study is thus the parametric counterpart of [Assa et al. \(2017\)](#), sharing its loss generating process.

Hazard (temperature). For each region r , the seasonal night-minimum temperature T_I is modelled as Gaussian with region-specific parameters and independent across seasons. In the loss model below this T_I plays the role of the night minimum, and the sunset/average temperature is reconstructed as $T_0 = T_I + N$ via the fitted nighttime drop N . Since the case study is simulation-based, the three regions are defined by their night-minimum laws (degrees Fahrenheit): $R1 \sim \mathcal{N}(25.5, 2.6^2)$, $R2 \sim \mathcal{N}(26.5, 2.5^2)$, $R3 \sim \mathcal{N}(27.5, 2.4^2)$, positioned around the citrus damaging band so that the single identical damage function is exercised with graded severity. The nighttime drop is taken from

the published January fit, $N \sim \mathcal{N}(9.41, 1.41^2)$ in degrees Fahrenheit (Assa et al., 2017). The Gaussian assumption is used for transparency; any alternative calibrated marginal can be substituted without changing the NatCat–NatPar logic.

Remark 3 (Where the catastrophe nature lives, and tail dependence under Gaussian margins). *The Gaussian hazard is light-tailed, so it carries no catastrophe character on its own. This is deliberate, and it locates the catastrophe nature of the loss downstream of the index, in two channels. First, the loss $L_r = A_r D(T_r)$ inherits the heavy right tail of the lognormal exposure A_r whenever $D_r > 0$; the unbounded severity that the bounded parametric payout cannot follow is exposure-driven, not hazard-driven. Second, the vulnerability map $D(\cdot)$ is sharply convex near the cold tail, so even a symmetric, light-tailed T_r produces a highly skewed damage variable D_r (a mass near zero plus a spike near one). The framework is marginal-agnostic: a genuinely heavy-tailed or clustered hazard—a generalised-extreme-value or generalised-Pareto fit to block minima, a skew- t , or a regime-switching process—can replace the Gaussian marginal without altering any of the reporting or pricing logic, and would simply add a third, hazard-side source of tail severity.*

Crucially, tail dependence is a separate matter from the marginal. By Sklar’s theorem the joint law factors into marginals and a copula, and the coefficient of tail dependence is a property of the copula alone. One may therefore impose joint cold extremes among Gaussian-margined temperatures without making any single region’s temperature heavy-tailed. The one-factor model of Section 9.3 couples the Gaussian margins with a Gaussian copula, which has zero tail dependence for every $\rho < 1$; the experiment of Section 9.4 keeps the same Gaussian margins but replaces the copula with a t -copula, so that $\lambda_L > 0$ while each regional temperature remains exactly Gaussian. The deep-tail relocation studied there is thus driven by the dependence structure, not by the marginal tails, and is well-posed despite the light-tailed hazard.

Exposure (value at risk). Exposure is modelled as lognormal, $A_r \sim \text{LogNormal}(\mu_{A,r}, \sigma_{A,r}^2)$, $A_r > 0$. Rather than parameterising by $(\mu_{A,r}, \sigma_{A,r})$ directly, we specify by region the mean $\bar{A}_r := \mathbb{E}[A_r]$ and the coefficient of variation $\text{CV}_{A,r} := \sqrt{\text{Var}(A_r)}/\bar{A}_r$, which implies $\sigma_{A,r}^2 = \ln(1 + \text{CV}_{A,r}^2)$ and $\mu_{A,r} = \ln(\bar{A}_r) - \frac{1}{2}\sigma_{A,r}^2$. We set $\bar{A}_r = 100$ for all regions and $\text{CV}_{A,r} = (0.60, 0.45, 0.50)$ for (R1, R2, R3), encoding region-specific exposure uncertainty. As a simplifying baseline we assume $T_r \perp A_r$.

Vulnerability and crop resistance. For the damage model we adopt the frost-loss construction of Assa et al. (2017), which combines the convex vulnerability curve of Venner et al. (1995) with the square-root nighttime-cooling forecast of Snyder & de Melo-Abreu (2005) to capture two features specific to frost: the damage is a *non-linear* function of temperature, and the crop *resists* the first few hours of freezing. Let T_I be the night-minimum temperature and T_0 the sunset (here, average) temperature. The Venner damage fraction on the night minimum is

$$D(T_I) = \begin{cases} 0, & T_I > T_c, \\ \left(\frac{T_c - T_I}{T_c - T_t}\right)^\eta, & T_t \leq T_I \leq T_c, \\ 1, & T_I < T_t, \end{cases}$$

with the citrus parameters $T_c = 28^\circ\text{F}$, $T_t = 20^\circ\text{F}$ (band width $b = T_c - T_t = 8^\circ\text{F}$), and curvature $\eta = 1.5$. Crop resistance enters through the requirement that the temperature stay k hours below T_c before damage occurs ($k = 4$ for citrus). Under the square-root cooling $T_i = T_0 + (T_I - T_0)\sqrt{i/I}$ over a night of length I hours, this resistance condition becomes $M \leq B N$, where $M = T_0 - T_c$,

$N = T_0 - T_I$ are the (independent) warm-margin and nighttime drop and $B = \sqrt{(I - k)/I}$. Realised damage is therefore

$$D = D(T_I) \mathbf{1}\{M \leq B N, M \leq N \leq M + b\},$$

so damage is active only when the night minimum lies in the damaging band *and* the crop's k -hour resistance is overcome. We follow Assa et al. (2017) in treating M and N as independent; the analytic loss CDF of that paper carries over, and we evaluate the resulting loss distribution by simulation here. The citrus band (T_c, T_t) , the curvature η , the band width b , and the entire resistance/ M – N machinery are held *identical across all three regions*—this is exactly the published damage function, unrelocated. The regions differ only in their night-minimum temperature law, so any difference in their loss distributions is attributable to the hazard, not to the vulnerability curve. Figure 2 shows the single shared vulnerability curve.

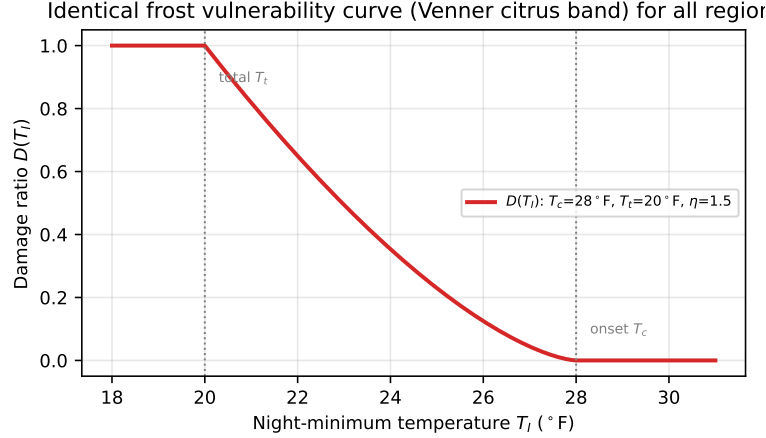


Figure 2: The single, identical frost vulnerability curve $D(T_I)$ used for all three regions, following the Venner et al. (1995) convex damage model with the published citrus parameters $T_c = 28^\circ\text{F}$, $T_t = 20^\circ\text{F}$ (band width $b = 8^\circ\text{F}$) and curvature $\eta = 1.5$. Damage is further gated by the Snyder & de Melo-Abreu (2005) k -hour resistance condition $M \leq B N$ as in Assa et al. (2017). The regions differ only in their night-minimum temperature law, not in this curve.

7.2 Basis risk diagnostics

This subsection compares the NatCat extreme-layer benchmark to two NatPar designs (continuous and binary) using exceedance and return-period diagnostics. Within each (r, β) block we price the trigger by the loss-minimisation of Section 5 under the symmetric-linear penalty, so the price is AAL-neutral and all differences reported below come from tail shape and basis structure, not from mean level. For each region r we simulate $N = 20,000$ seasons and compute damage and loss as $D_r = D(T_r) \in [0, 1]$ and $L_r = A_r D_r$. For each $\beta \in \{0.90, 0.95, 0.99\}$ we define the NatCat extreme-layer benchmark via $l_{r,\beta} := \text{VaR}_\beta(L_r)$, $Y_{r,\beta} := (L_r - l_{r,\beta})^+$.

7.2.1 Individual risk

We evaluate two NatPar archetypes, $P_{r,\beta}^c = q_{r,\beta}^c (D_r - d_{r,\beta}^c)^+$ and $P_{r,\beta}^b = q_{r,\beta}^b \mathbf{1}\{D_r > d_{r,\beta}^b\}$. Rather than pinning the trigger at an ad-hoc percentile, the triggers $d_{r,\beta}^{\{\cdot\}}$ are the *optimal* solutions of (1): for the continuous design the pair $(d_{r,\beta}^c, q_{r,\beta}^c)$ is chosen jointly to minimise the symmetric-linear basis loss; for the binary design the single trigger $d_{r,\beta}^b$ is optimised with $q_{r,\beta}^b$ pinned by AAL-neutrality.

Because the symmetric-linear penalty sits on the AAL-neutral corner (Remark 2), each design still satisfies $\mathbb{E}[P_{r,\beta}^{\{\cdot\}}] = \mathbb{E}[Y_{r,\beta}]$, so the comparison remains like-for-like in mean.

Optimal versus quantile triggers. To show that the optimisation does real work—and where it does not—Table 1 contrasts the optimal trigger with the natural alternative of pinning the trigger at the loss quantile, $d_{r,\beta} = \text{VaR}_\beta(D_r)$, holding both designs AAL-neutral so the comparison is like-for-like in mean and only the basis dispersion moves. Two patterns emerge. For the *continuous* design the optimal trigger sits systematically *below* the quantile pin and reduces basis variance by roughly 8–24%, the improvement growing with β : pinning the ramp at a high loss percentile starts it too late, leaving frequent moderate shortfalls that a lower, optimised onset removes. For the *binary* design the two triggers land close and the optimisation yields essentially no gain (and occasionally a slight loss in variance), because the single trigger is already pinned by the AAL budget—there is no genuine free degree of freedom for the loss-minimisation to exploit, exactly as the determinacy discussion of Section 5 anticipates. The practical reading is that trigger optimisation matters for graded (continuous) payouts and is close to a no-op for digital ones, so the value of pricing-as-loss-minimisation is realised precisely where the contract has spare shape to give.

Table 1: Optimal trigger versus the quantile pin $d = \text{VaR}_\beta(D)$, both AAL-neutral. Columns: quantile trigger d_{qtl} , optimal trigger d_{opt} , basis variance under each, and the percentage reduction from optimising. The optimisation reduces basis dispersion for the continuous design (c) but is essentially inert for the binary design (b), whose single trigger is pinned by the budget.

Region	β	Design	d_{qtl}	d_{opt}	$\text{Var}(B)_{\text{qtl}}$	$\text{Var}(B)_{\text{opt}}$	reduction %
R1	0.90	c	0.593	0.483	238.0	198.3	16.7
R1	0.90	b	0.593	0.600	212.5	214.7	-1.0
R1	0.95	c	0.720	0.591	157.2	124.0	21.1
R1	0.95	b	0.720	0.732	137.8	140.3	-1.8
R1	0.99	c	0.912	0.816	42.3	32.8	22.5
R1	0.99	b	0.912	0.862	38.8	33.7	13.1
R2	0.90	c	0.436	0.378	85.0	74.6	12.2
R2	0.90	b	0.436	0.499	84.1	89.1	-5.9
R2	0.95	c	0.557	0.488	60.9	51.4	15.6
R2	0.95	b	0.557	0.585	53.5	56.2	-5.0
R2	0.99	c	0.798	0.706	20.8	15.9	23.5
R2	0.99	b	0.798	0.800	17.5	17.5	-0.3
R3	0.90	c	0.302	0.261	67.7	62.4	7.8
R3	0.90	b	0.302	0.386	79.7	80.5	-0.9
R3	0.95	c	0.418	0.370	52.2	46.1	11.7
R3	0.95	b	0.418	0.465	50.5	52.7	-4.4
R3	0.99	c	0.648	0.543	22.2	17.0	23.4
R3	0.99	b	0.648	0.623	18.0	17.6	2.2

Exceedance probability curves. Figure 3 compares the exceedance probability functions on a log scale. The binary design places a point mass at $q_{r,\beta}^b$, so $\text{EP}^{b,+}(x)$ is essentially flat up to that level and drops to zero immediately after, creating the visible “digital” tail geometry relative to the smoother benchmark and continuous design.

Exceedance probability curves by region and β

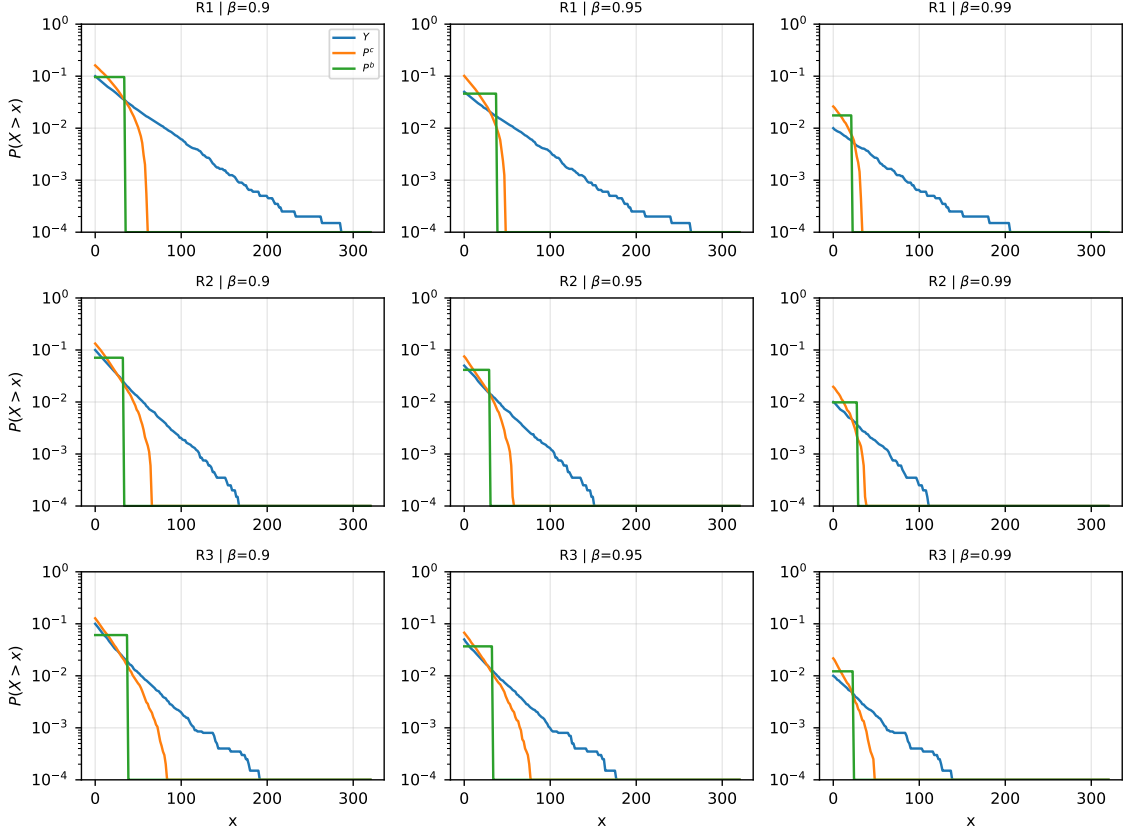


Figure 3: Exceedance probability curves for $Y_{r,\beta}$, $P_{r,\beta}^{\{\cdot\}}$ (log scale), by region and β .

Point diagnosis of basis risk. Table 2 reports the calibration levels and dependence diagnostics: the layer attachment $l_{r,\beta}$, the *optimal* damage trigger $d_{r,\beta}^{\{\cdot\}}$ and scale $q_{r,\beta}$, $\mathbb{E}[Y_{r,\beta}]$, $\text{Var}(B)$, and $\text{Corr}(Y_{r,\beta}, P_{r,\beta}^{\{\cdot\}})$. Table 3 reports the basis mismatch decomposition: $\mathbb{P}(B > 0)$, $\mathbb{P}(B < 0)$, average shortfall $\mathbb{E}[B^+]$, and average overpayment $\mathbb{E}[(-B)^+]$.

Table 2: Point diagnosis: calibration levels and dependence. The trigger d is the optimal solution of the symmetric-linear loss-minimisation; $l_{r,\beta} = \text{VaR}_\beta(L_r)$ is the layer attachment.

Region	β	Design	$l_{r,\beta}$	$d_{r,\beta}$	$q_{r,\beta}$	$\mathbb{E}[Y_\beta]$	$\text{Var}(B)$	$\text{Corr}(Y_\beta, P_\beta)$
R1	0.90	c	59.99	0.483	116.68	3.39	198.25	0.45
R1	0.90	b	59.99	0.600	35.17	3.39	214.67	0.40
R1	0.95	c	82.60	0.591	117.67	1.77	124.01	0.34
R1	0.95	b	82.60	0.732	38.43	1.77	140.34	0.31
R1	0.99	c	141.37	0.816	181.15	0.38	32.75	0.15
R1	0.99	b	141.37	0.862	21.48	0.38	33.71	0.13
R2	0.90	c	42.57	0.378	106.57	2.39	74.64	0.61
R2	0.90	b	42.57	0.499	33.73	2.39	89.07	0.54
R2	0.95	c	58.46	0.488	111.08	1.25	51.40	0.49
R2	0.95	b	58.46	0.585	30.00	1.25	56.16	0.45
R2	0.99	c	97.87	0.706	128.04	0.27	15.93	0.24
R2	0.99	b	97.87	0.800	27.86	0.27	17.52	0.22
R3	0.90	c	28.84	0.261	113.75	2.27	62.42	0.67
R3	0.90	b	28.84	0.386	37.42	2.27	80.47	0.59
R3	0.95	c	43.72	0.370	123.35	1.21	46.06	0.57
R3	0.95	b	43.72	0.465	32.81	1.21	52.75	0.51
R3	0.99	c	82.03	0.543	106.42	0.29	17.02	0.34
R3	0.99	b	82.03	0.623	23.93	0.29	17.58	0.32

Table 3: Point diagnosis: basis mismatch decomposition.

Region	β	Design	$\mathbb{P}(P > 0)$	$\mathbb{P}(B > 0)$	$\mathbb{P}(B < 0)$	$\mathbb{E}[B^+]$	$\mathbb{E}[(-B)^+]$
R1	0.90	c	0.16	0.07	0.12	2.09	2.09
R1	0.90	b	0.10	0.07	0.07	2.16	2.16
R1	0.95	c	0.10	0.04	0.08	1.28	1.28
R1	0.95	b	0.05	0.04	0.04	1.29	1.29
R1	0.99	c	0.03	0.01	0.02	0.33	0.33
R1	0.99	b	0.02	0.01	0.02	0.34	0.34
R2	0.90	c	0.13	0.06	0.09	1.16	1.16
R2	0.90	b	0.07	0.07	0.05	1.25	1.25
R2	0.95	c	0.08	0.03	0.06	0.74	0.74
R2	0.95	b	0.04	0.04	0.03	0.77	0.77
R2	0.99	c	0.02	0.01	0.02	0.22	0.22
R2	0.99	b	0.01	0.01	0.01	0.22	0.22
R3	0.90	c	0.13	0.06	0.09	1.02	1.02
R3	0.90	b	0.06	0.07	0.04	1.13	1.13
R3	0.95	c	0.07	0.03	0.05	0.66	0.66
R3	0.95	b	0.04	0.03	0.03	0.69	0.69
R3	0.99	c	0.02	0.01	0.02	0.22	0.22
R3	0.99	b	0.01	0.01	0.01	0.22	0.22

Basis exceedance curves. Figure 4 reports the two-sided basis exceedance functions, separating consumer-protection tail risk (shortfall) from capital/pricing tail risk (overpayment). The binary design exhibits a sharper truncation in the overpayment tail (due to the fixed payout size), while the continuous design decays more smoothly.

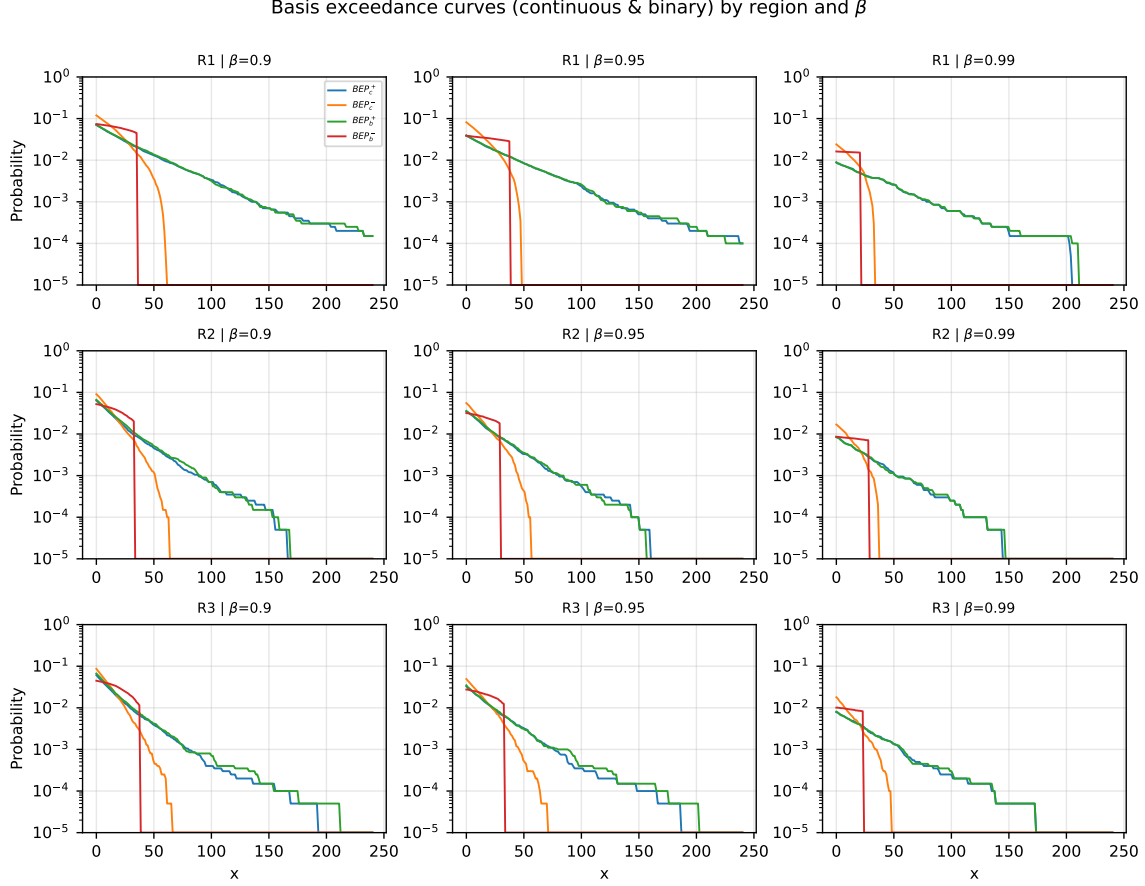


Figure 4: Basis exceedance curves for continuous and binary NatPar by region and β (log scale).

Return-period levels. A key feature in the results is the presence of many zeros at low return periods: for the layer $Y_{r,\beta} = (L - l_{r,\beta})^+$, we have $\mathbb{P}(Y_{r,\beta} > 0) = 1 - \beta$, so when $1/T \geq 1 - \beta$ the return level is exactly zero. The binary payout also produces plateau behaviour in $x_T(P_{r,\beta}^b)$ because of its point mass at $q_{r,\beta}^b$; see Figures 5 and 6.

Return-period curves for liabilities by region and β

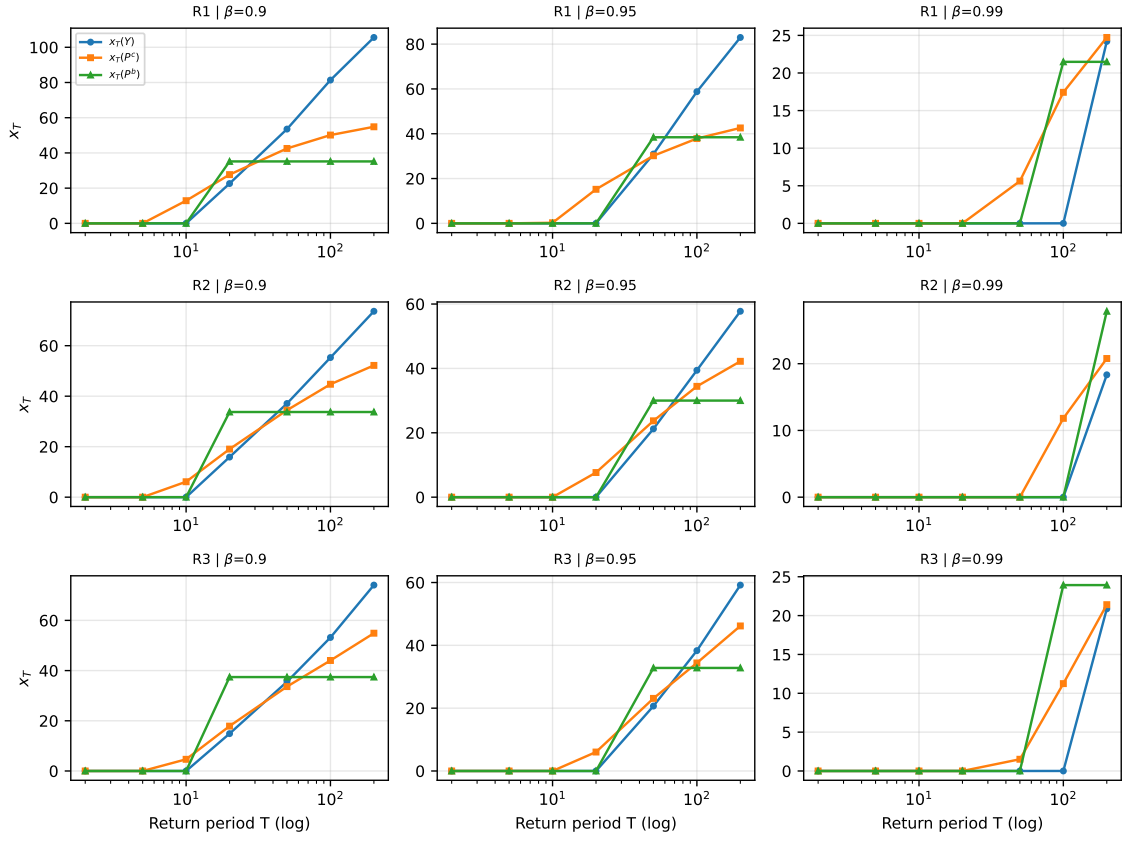


Figure 5: Return-period curves $T \mapsto x_T(\cdot)$ for $Y_{r,\beta}$, $P_{r,\beta}^{\{\cdot\}}$, by region and β .

Return-period curves for basis tails by region and β

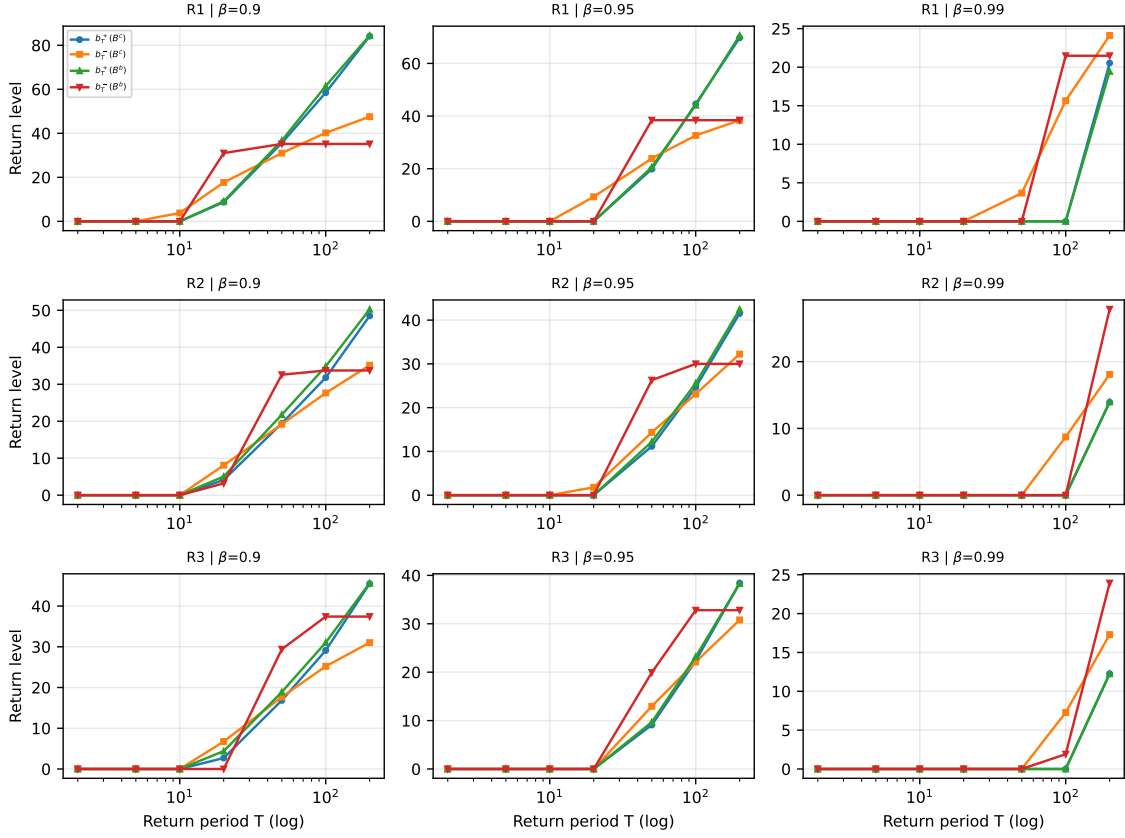


Figure 6: Return-period curves for basis tails: $T \mapsto b_T^+$ (shortfall) and $T \mapsto b_T^-$ (overpayment), for both continuous and binary NatPar, by region and β .

Capital metrics. Table 4 reports $\text{VaR}_{0.995}$ and $\text{TVaR}_{0.995}$ for $Y_{r,\beta}$ and each NatPar design across β . In this case-study setting (at most one seasonal event per year) these correspond directly to EP/AEP return levels at exceedance probability 0.5%.

Table 4: Capital metrics for NatCat layer $Y_{r,\beta}$ and NatPar liabilities $P_{r,\beta}^{\{\cdot\}}$: $\text{VaR}_{0.995}$ and $\text{TVaR}_{0.995}$, by region and layer β .

Region	β	$\text{VaR}(Y_\beta)$	$\text{TVaR}(Y_\beta)$	$\text{VaR}(P_\beta^c)$	$\text{TVaR}(P_\beta^c)$	$\text{VaR}(P_\beta^b)$	$\text{TVaR}(P_\beta^b)$
R1	0.90	105.58	145.35	54.84	57.29	35.17	35.17
R1	0.95	82.97	122.75	42.59	45.06	38.43	38.43
R1	0.99	24.20	63.97	24.72	28.52	21.48	21.48
R2	0.90	73.63	101.91	52.19	58.09	33.73	33.73
R2	0.95	57.75	86.02	42.21	48.35	30.00	30.00
R2	0.99	18.33	46.61	20.78	27.87	27.86	27.86
R3	0.90	74.06	102.57	54.89	64.53	37.42	37.42
R3	0.95	59.18	87.69	46.17	56.62	32.81	32.81
R3	0.99	20.87	49.37	21.41	30.42	23.93	23.93

7.2.2 How the penalty shape moves the price

We now solve the loss-minimisation (1) for the continuous design at a representative layer $\beta = 0.95$ in each region, sweeping a panel of penalties that vary the weight λ , the curvature exponents (p_-, p_+) , and—in the last row—swap the power penalty for an exponential shortfall loss $\phi_-(z) = e^{\kappa z} - 1$ with linear over-pay. Table 5 reports the calibrated trigger d , scale q , mean payout $\mathbb{E}[P]$, and the realised mean basis $\mathbb{E}[B]$; Table 6 reports the shortfall $\mathbb{E}[B^+]$, over-pay $\mathbb{E}[(-B)^+]$, dependence, hedging effectiveness $HE = 1 - \text{Var}(L - P)/\text{Var}(L)$, and the shortfall capital $\text{VaR}_{0.995}(B^+)$. A soft budget ($\mu = 3$) keeps the premium anchored unless the curvature deliberately overrides it.

Three patterns stand out. First, *weight alone barely moves the price*: the symmetric and shortfall-tilted L_1 rows are nearly identical in (d, q) and sit on $\mathbb{E}[B] = 0$, because under a linear penalty the scale direction is governed by the mean-basis condition of Remark 2 regardless of λ . Second, *curvature moves it decisively*: raising the shortfall exponent from 1 to 2 to 3 drives the scale q up sharply, pushes $\mathbb{E}[P]$ above the AAL, and collapses the shortfall tail— $\text{VaR}_{0.995}(B^+)$ falls from roughly 45 at (1, 1) to near zero at (3, 1) in every region—while the over-pay $\mathbb{E}[(-B)^+]$ balloons correspondingly. This is the shortfall/over-pay trade-off of Figure 7, traced by curvature rather than by weight. Third, *the general loss case behaves as expected*: the exponential shortfall penalty produces an intermediate, region-dependent price that targets extreme under-payment without the explosive scale of the cubic power penalty. Symmetric curvature (2, 2) leaves the price essentially AAL-neutral, confirming that it is the *asymmetry* of curvature, not its presence, that relocates the premium.

Table 5: Penalty shape and the price (continuous design, $\beta = 0.95$). Columns: weight λ , curvature (p_-, p_+) , trigger d , scale q , mean payout $\mathbb{E}[P]$, mean basis $\mathbb{E}[B]$.

Region	Penalty	λ	(p_-, p_+)	d	q	$\mathbb{E}[P]$	$\mathbb{E}[B]$
R1	L_1 symmetric	0.50	(1,1)	0.591	117.67	1.77	0.000
R1	L_1 short-tilt	0.80	(1,1)	0.591	117.85	1.77	0.000
R1	quad short	0.80	(2,1)	0.435	243.63	9.14	-7.365
R1	quad both	0.50	(2,2)	0.399	39.30	1.77	-0.000
R1	cubic short	0.80	(3,1)	0.179	427.32	51.34	-49.567
R1	exp short	0.80	exp	0.709	432.80	2.68	-0.902
R2	L_1 symmetric	0.50	(1,1)	0.488	111.08	1.25	-0.000
R2	L_1 short-tilt	0.80	(1,1)	0.488	111.12	1.25	-0.000
R2	quad short	0.80	(2,1)	0.387	177.06	3.79	-2.544
R2	quad both	0.50	(2,2)	0.369	52.40	1.25	-0.000
R2	cubic short	0.80	(3,1)	0.138	289.01	23.34	-22.094
R2	exp short	0.80	exp	0.512	130.85	1.25	0.000
R3	L_1 symmetric	0.50	(1,1)	0.370	123.35	1.21	0.000
R3	L_1 short-tilt	0.80	(1,1)	0.370	123.39	1.21	-0.000
R3	quad short	0.80	(2,1)	0.310	242.48	3.54	-2.329
R3	quad both	0.50	(2,2)	0.294	74.66	1.21	0.000
R3	cubic short	0.80	(3,1)	0.185	454.17	14.67	-13.462
R3	exp short	0.80	exp	0.523	377.22	1.21	0.000

Table 6: Penalty shape and the basis (continuous design, $\beta = 0.95$). Shortfall $\mathbb{E}[B^+]$, over-pay $\mathbb{E}[(-B)^+]$, dependence, hedging effectiveness HE , and shortfall capital $\text{VaR}_{0.995}(B^+)$.

Region	Penalty	$\mathbb{E}[B^+]$	$\mathbb{E}[(-B)^+]$	Corr	HE	$\text{VaR}_{0.995}(B^+)$
R1	L_1 symmetric	1.28	1.28	0.34	0.060	69.72
R1	L_1 short-tilt	1.28	1.28	0.34	0.060	69.72
R1	quad short	0.53	7.90	0.36	-2.912	40.52
R1	quad both	1.34	1.34	0.36	0.131	70.54
R1	cubic short	0.01	49.58	0.34	-42.709	0.00
R1	exp short	1.19	2.09	0.30	-0.731	68.72
R2	L_1 symmetric	0.74	0.74	0.49	0.184	41.52
R2	L_1 short-tilt	0.74	0.74	0.49	0.184	41.51
R2	quad short	0.33	2.87	0.51	-1.089	21.73
R2	quad both	0.77	0.77	0.51	0.257	43.32
R2	cubic short	0.01	22.11	0.45	-24.590	0.00
R2	exp short	0.74	0.74	0.48	0.143	40.73
R3	L_1 symmetric	0.66	0.66	0.57	0.292	38.41
R3	L_1 short-tilt	0.66	0.66	0.57	0.292	38.41
R3	quad short	0.28	2.61	0.58	-1.269	18.67
R3	quad both	0.68	0.68	0.58	0.332	39.46
R3	cubic short	0.02	13.48	0.55	-21.318	0.00
R3	exp short	0.75	0.75	0.49	-0.308	40.03

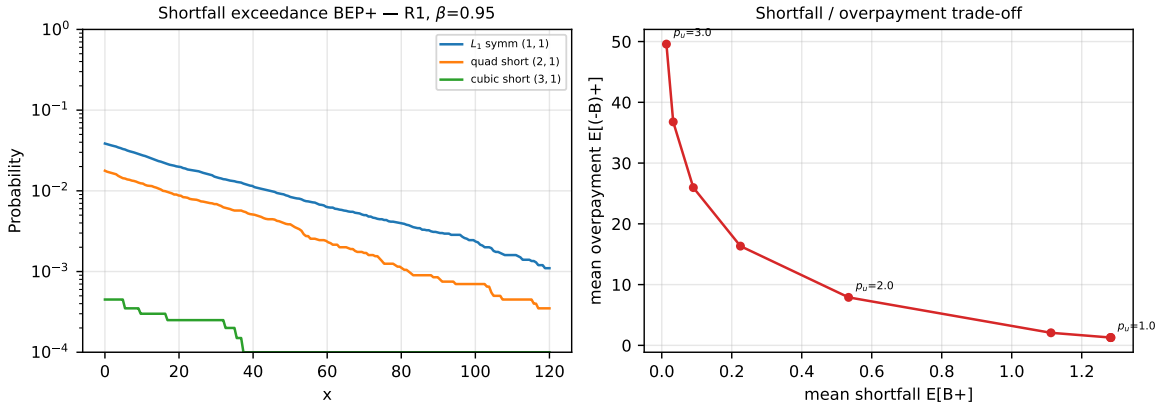


Figure 7: Penalty curvature shapes the price (continuous design, R1, $\beta = 0.95$). Left: the shortfall exceedance curve BEP^+ thins as the shortfall exponent rises from 1 to 3. Right: the resulting shortfall/over-pay trade-off, traced as the shortfall exponent p_- sweeps from 1 to 3 at fixed weight—heavier curvature buys a thinner shortfall tail by accepting more over-payment.

7.2.3 Portfolio risk

The numerical assessment is based on Monte Carlo samples of regional indemnity losses $L_r = A_r D(T_r)$. Within each region, A_r and T_r are independent. At the portfolio level, regional sample pairs are combined by aligning draws across regions; unless stated otherwise this treats the regional

pairs as independent. Let $B_{r,\beta}^{\{\cdot\}} = Y_{r,\beta} - P_{r,\beta}^{\{\cdot\}}$ denote the basis in region r at layer β . The aggregate and occurrence basis are $S_\beta^B = \sum_{r \in \mathcal{R}} B_{r,\beta}^{\{\cdot\}}$ and $M_\beta^B = \max_{r \in \mathcal{R}} B_{r,\beta}^{\{\cdot\}}$.

AEP and OEP for indemnity and parametric portfolio

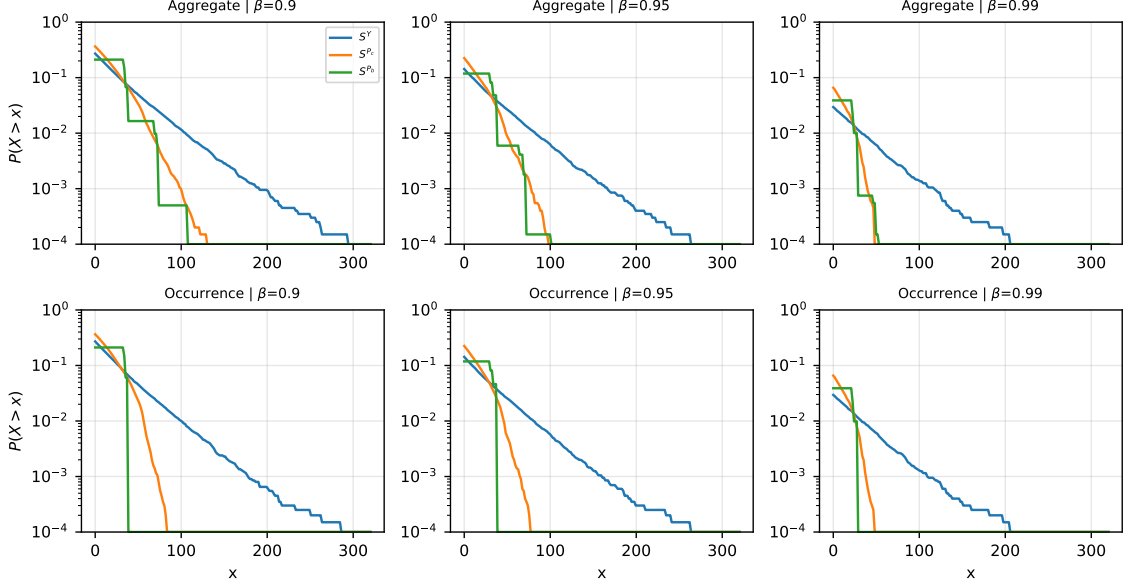


Figure 8: AEP and OEP for the indemnity and parametric portfolio.

Tables 7 and 8 report the aggregate-vs-occurrence level/dependence and point diagnosis. Table 9 reports portfolio capital metrics. Figures 9, 10, and 11 report the basis exceedance and return-period curves.

Table 7: Aggregate vs Occurrence: level and dependence.

Scope	Design	β	$\mathbb{E}[Y]$	$\text{Corr}(Y, P)$	$\text{Var}(B)$
Agg	b	0.90	8.05	0.49	386.64
Agg	b	0.95	4.23	0.40	250.97
Agg	b	0.99	0.94	0.22	70.18
Agg	c	0.90	8.05	0.55	338.01
Agg	c	0.95	4.23	0.45	224.02
Agg	c	0.99	0.94	0.24	67.12
Occ	b	0.90	7.68	0.47	221.56
Occ	b	0.95	4.12	0.38	152.09
Occ	b	0.99	0.93	0.22	49.76
Occ	c	0.90	7.68	0.53	207.35
Occ	c	0.95	4.12	0.44	147.02
Occ	c	0.99	0.93	0.23	48.81

Table 8: Aggregate vs Occurrence: point diagnosis.

Scope	Design	β	$\mathbb{P}(P > 0)$	$\mathbb{P}(B > 0)$	$\mathbb{P}(B < 0)$	$\mathbb{E}[B^+]$	$\mathbb{E}[(-B)^+]$
Agg	b	0.90	0.21	0.18	0.15	4.24	4.24
Agg	b	0.95	0.12	0.10	0.09	2.64	2.64
Agg	b	0.99	0.04	0.02	0.03	0.76	0.76
Agg	c	0.90	0.36	0.17	0.25	3.95	3.95
Agg	c	0.95	0.22	0.10	0.17	2.55	2.55
Agg	c	0.99	0.07	0.02	0.06	0.76	0.76
Occ	b	0.90	0.21	0.19	0.00	4.40	0.00
Occ	b	0.95	0.12	0.11	0.00	2.70	0.00
Occ	b	0.99	0.04	0.03	0.00	0.77	0.00
Occ	c	0.90	0.36	0.18	0.00	4.14	0.01
Occ	c	0.95	0.22	0.10	0.00	2.63	0.00
Occ	c	0.99	0.07	0.02	0.00	0.77	0.00

BAEP and BOEP for continuous and binary parametric portfolio

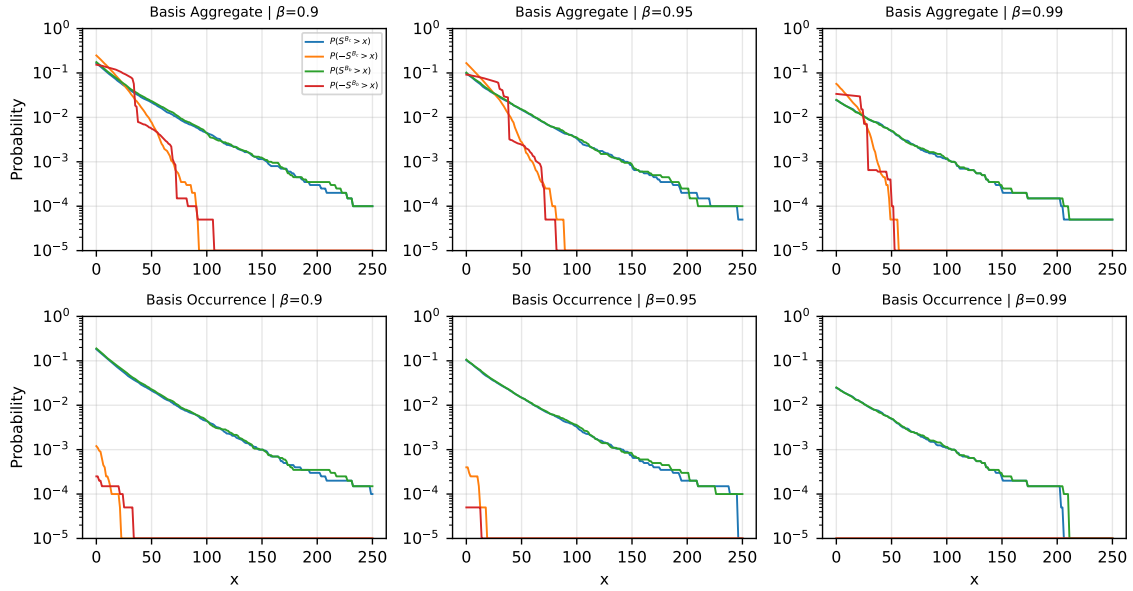


Figure 9: BAEP and BOEP for the continuous and binary parametric portfolio.

Return-period curves for liabilities (Aggregate vs Occurrence)

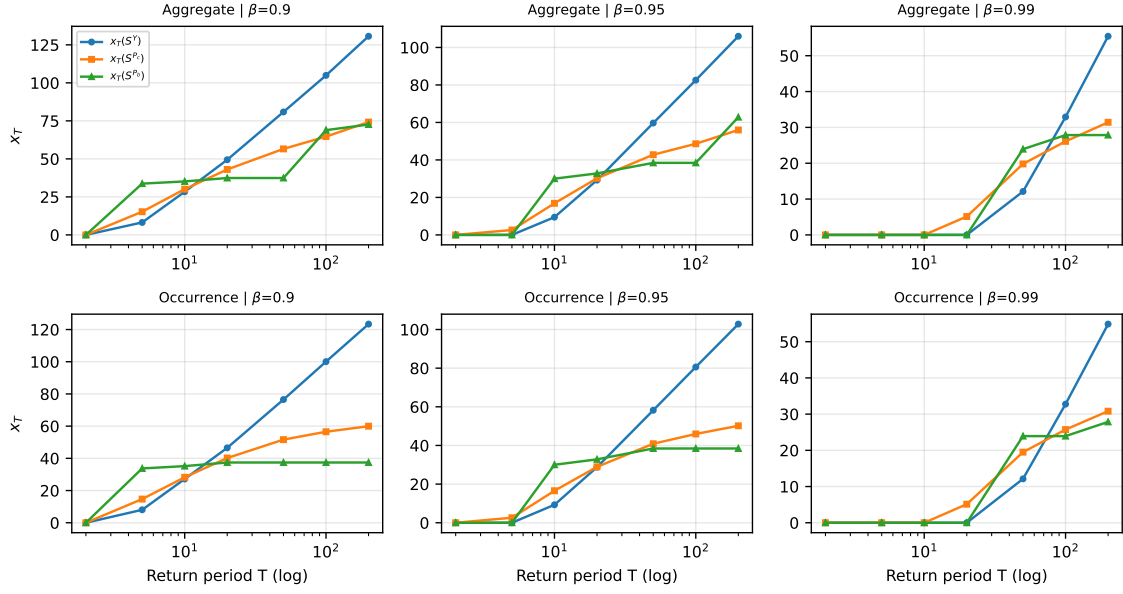


Figure 10: Return-period curves for liabilities (Aggregate vs Occurrence).

Return-period curves for basis (Aggregate vs Occurrence)

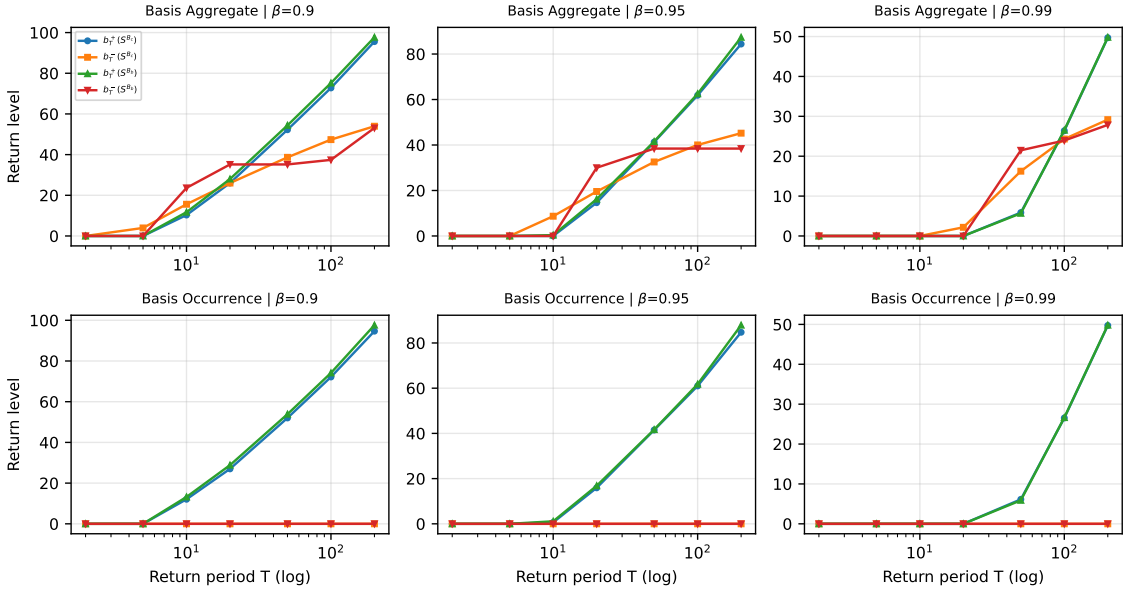


Figure 11: Return-period curves for basis (Aggregate vs Occurrence, continuous & binary, $\pm$).

Table 9: Capital metrics for portfolio of NatCat layer $Y_{r,\beta}$ and NatPar liabilities $P_{r,\beta}^{\{\cdot\}}$: $\text{VaR}_{0.995}$ and $\text{TVaR}_{0.995}$, by layer β .

Scope	β	$\text{VaR}(Y_\beta)$	$\text{TVaR}(Y_\beta)$	$\text{VaR}(P_\beta^c)$	$\text{TVaR}(P_\beta^c)$	$\text{VaR}(P_\beta^b)$	$\text{TVaR}(P_\beta^b)$
Agg	0.90	130.65	169.28	74.23	88.68	72.59	75.50
Agg	0.95	105.98	145.01	55.97	69.71	62.81	68.29
Agg	0.99	55.43	91.02	31.42	36.54	27.86	29.42
Occ	0.90	123.37	158.98	59.96	66.77	37.42	37.42
Occ	0.95	102.80	138.95	50.17	58.49	38.43	38.43
Occ	0.99	54.86	89.54	30.81	35.20	27.86	27.86

8 Results interpretation of the frost case study

Figures 3–4 reveal a systematic pattern: in the far tail, the shortfall component of basis risk dominates the overpayment component. Interpreting this correctly requires separating an economic conclusion (who benefits) from a mathematical one (why the tail geometry looks the way it does). The plots primarily reflect a structural feature of the design: the indemnity-layer benchmark $Y_{r,\beta}$ is driven by exposure–severity through $L = AD$, while the parametric payouts are functions of the damage index D alone and are therefore bounded.

The core mechanism. In our setting $L = A_r D_r$, where the random exposure/severity multiplier $A_r > 0$ is independent of D_r and may exhibit substantial right-tail variability, while the damage ratio satisfies $D_r \in [0, 1]$. The NatPar designs are functions of D and inherit its bounded support: $0 \leq P_{r,\beta}^b \leq q_{r,\beta}^b$ and $0 \leq P_{r,\beta}^c \leq q_{r,\beta}^c(1 - d_{r,\beta}^{\{\cdot\}})$, where $d_{r,\beta}^{\{\cdot\}}$ is the optimal trigger and $q_{r,\beta}^{\{\cdot\}}$ the AAL-neutral scale that matches the mean layer loss $\mathbb{E}[Y_{r,\beta}]$. In contrast, $Y_{r,\beta} = (A_r D_r - l_{r,\beta})^+$ can be large whenever A_r is large enough, even if D_r is only moderate. Consequently, in extreme states the basis behaves as $B_{r,\beta}^{\{\cdot\}} \approx Y_{r,\beta}$ whenever $Y_{r,\beta} \gg \sup P_{r,\beta}^{\{\cdot\}}$, so the far-right tail of $B_{r,\beta}^{\{\cdot\}}$ inherits the tail of Y_β while the overpayment side is capped. This is the mathematical reason the return-period shortfall curves grow faster than the corresponding overpayment curves in Figure 6, and why the basis exceedance curves BEP^+ remain comparatively heavy in Figure 4.

Unspanned severity. The dominance of shortfall is not merely “because P is bounded”; it arises because a D -only payout cannot span exposure-driven severity. Large values of $Y_{r,\beta}$ can occur via two mechanisms: (i) A_r large with moderate D_r , or (ii) D_r large with moderate A_r . Binary and continuous NatPar designs respond only to mechanism (ii). Under independence of A and D , mechanism (i) occurs with non-negligible probability in the tail, producing states where $Y_{r,\beta} > 0$ but $P_{r,\beta}^{\{\cdot\}}$ is small or zero, hence $B_{r,\beta}^{\{\cdot\}}$ is large and positive (shortfall). Overpayment requires the reverse mismatch: D large together with small A_r so that $Y_{r,\beta}$ is near zero while $P_{r,\beta}^{\{\cdot\}}$ is positive. Such states exist, but their severity is capped by $\sup P_{r,\beta}^{\{\cdot\}}$, so overpayment does not dominate at long return periods.

Profitability. It is tempting to read “shortfall dominates overpayment” as an issuer advantage and therefore a reason to underwrite the product. That inference is incomplete. Overpayment corresponds to paying more than the benchmark layer and is the issuer’s direct economic downside relative to $Y_{r,\beta}$. Shortfall, however, is not a cash loss to the issuer; it is a coverage failure borne

by the policyholder and often a commercial or conduct risk for the issuer. A design that generates large tail shortfalls may be capital-tractable (bounded liability) but can be commercially fragile. Hence tail shortfall dominance is primarily evidence of bounded-liability structure rather than a direct profitability statement.

Design implication. We select $q_{r,\beta}^{\{\cdot\}}$ to match $\mathbb{E}[P_{r,\beta}^{\{\cdot\}}] = \mathbb{E}[Y_{r,\beta}]$ (AAL matching at the layer). Figures 3 and 5 show that this mean matching can coexist with large differences in tail behaviour, because matching $\mathbb{E}[Y_{r,\beta}]$ does not control how Y_β decomposes into exposure-driven versus damage-driven extremes. When exposure variability dominates severity, a damage-only index necessarily under-replicates the far tail.

Individual risk: what the three regions show. The individual capital table (Table 4) shows that the three regional assets behave differently in the far tail and that the difference is design-dependent. For the most frost-exposed region R1 at $\beta = 0.90$, the benchmark layer remains heavy-tailed with $\text{VaR}_{0.995}(Y_\beta) \approx 106$ and $\text{TVaR}_{0.995}(Y_\beta) \approx 145$, while the optimally-triggered continuous design tracks the body but caps the tail ($\text{VaR}_{0.995}(P_\beta^c) \approx 55$, $\text{TVaR}_{0.995}(P_\beta^c) \approx 57$) and the binary design is more strongly capped ($\text{VaR}_{0.995}(P_\beta^b) \approx 35$). The graded severity across regions—R1 heaviest, then R3 and R2—is produced entirely by the hazard law, since the damage curve is identical. At $\beta = 0.99$ the loss layer is thin (e.g. $\text{VaR}_{0.995}(Y_\beta) \approx 18$ for R2), and the digital binary payout, sized to be AAL-neutral against that thin layer, can carry a liability *as large as* the indemnity layer’s tail capital and larger than the continuous design’s ($\text{VaR}_{0.995}(P_\beta^b) \approx 28$ for R2, against $\text{VaR}_{0.995}(Y_\beta) \approx 18$ and $\text{VaR}_{0.995}(P_\beta^c) \approx 21$): the point mass lands near the layer’s 99.5th percentile. This is the capital cost of digitisation made explicit, and it is a feature of payout shape, not of mean level. The optimal trigger keeps all three regions responsive at high β —the continuous design tracks the layer closely ($\text{VaR}_{0.995}(P_\beta^c) \approx 21$ – 25 against $\text{VaR}_{0.995}(Y_\beta) \approx 18$ – 24 across regions)—so optimisation avoids the tail non-spanning that an arbitrary high-percentile trigger would have produced. Because the damage curve is identical across regions, these differences isolate the effect of the hazard law on the tail.

Portfolio versus individual risk. Once we move to the three-asset portfolio, the point-diagnosis tables clarify what pooling does and does not fix. First, the mean layer size drops sharply as β increases ($\mathbb{E}[S^Y] \approx 9.0$ at $\beta = 0.90$, 4.9 at $\beta = 0.95$, and 1.2 at $\beta = 0.99$); this is the mechanical effect of pushing the attachment deeper into the tail, not a diversification effect. The portfolio story is instead in dependence and mismatch. The continuous design remains systematically more aligned with the benchmark than the binary design: in the aggregate (AEP-style) view, $\text{Corr}(S^Y, S^P)$ is about 0.71 for design *c* versus 0.62 for design *b* at $\beta = 0.90$, narrowing toward $0.33/0.32$ as $\beta \rightarrow 0.99$ where both designs become sparse. More importantly for governance, the portfolio basis dispersion is higher for the binary design: $\text{Var}(S^B)$ is about 397 (*b*) versus 306 (*c*) at $\beta = 0.90$, and the occurrence dispersion $\text{Var}(M^B)$ is about 238 (*b*) versus 198 (*c*). This is the numerical counterpart of what the basis-tail plots suggest: the binary contract is not just capped, it is capped in a way that concentrates the unspanned component into the shortfall side whenever exposure-driven severity dominates.

The same conclusion holds under the occurrence (OEP-style) lens. The occurrence view is the one that bites for suitability and operational stress: even if the aggregate mismatch nets out across assets in some years, a single region can still experience a large shortfall. That is why, in addition to reporting the aggregate basis $S^B = \sum_r B_r$, we recommend an occurrence basis $M^B := \max_r B_r$ as a regulatory object. Importantly, this definition should be based on the per-asset basis terms

(not portfolio-level quantiles of L and D), because attachment and triggers are contractual at the risk level; computing “portfolio” l_β^{port} and d_β^{port} mixes attachment/triggers across assets and can mask the very cross-asset tail asymmetries the portfolio analysis is meant to reveal. Overall, the portfolio does not reverse the individual diagnosis; it sharpens it.

Pricing implication. Tables 5–6 add the pricing lens to this picture. The premium is governed far more by the *curvature* of the mismatch penalty than by its weight: shortfall-tilting the weight under a linear penalty leaves the price on the AAL-neutral value, whereas convex shortfall curvature lifts the scale and thins the shortfall tail at the cost of overpayment, tracing the trade-off in Figure 7. The AAL-neutral (“fair”) price is recovered only at the symmetric-linear corner; it is a useful reference point, not a distinct objective. For binary triggers the single trigger is pinned by the budget alone, so curvature has no purchase—a useful guardrail against over-claiming pricing flexibility for digital designs.

Regulatory and reporting implication. The basis diagnostics provide information that cannot be inferred from AAL and standard EP curves alone. Basis exceedance curves $\text{BEP}^\pm$ summarise the frequency of extreme shortfall/overpayment events as a function of severity, while basis return levels $b_T^\pm$ translate those exceedance probabilities into T -year basis severities. These make explicit whether a parametric design is best interpreted as a loss proxy with acceptable tail mismatch, or as a bounded liquidity instrument whose tail protection is limited by construction. We therefore view the pair $(\text{BEP}^\pm, b_T^\pm)$ as essential complements to the usual NatCat objects.

Market benefit: liquidity. A practical benefit of the bounded, index-linked structure is not that it perfectly replicates tail loss, but that it can function as a contingent liquidity instrument in markets where high exposure uncertainty creates insurability gaps. When exposure variability is large or hard to verify quickly, indemnity-style settlement becomes slow and costly. A parametric contract tied to an observable hazard/damage index D can still be offered because the liability is auditable and capped. The relevant question is not whether BEP^+ can be made negligible, but whether the trigger and scale deliver timely cash in the states where liquidity constraints bind, and whether residual loss risk is transparently disclosed and, where needed, complemented by additional layers.

9 Who gains, and when: a temporal reading of basis risk

The basis return-period curves of Section 7 support an economic reading that, to our knowledge, is not made explicit in the design-first literature: under an AAL-neutral parametric contract, insurer and insured are not made indifferent but are placed on *opposite sides of the time axis*. This section develops that reading and connects it to why NatPar has grown in popularity as catastrophe risk has intensified.

9.1 The crossover horizon

Fix the basis $B = Y - P$, so $B > 0$ is shortfall (the insured is under-covered) and $B < 0$ is over-payment (the insurer pays more than the realised loss). Under the symmetric-linear optimal trigger the mean basis is zero and the two average severities coincide ($\mathbb{E}[B^+] = \mathbb{E}[B^-]$), so over a full statistical lifetime neither party has an edge—this is exactly the content of Remark 1. The asymmetry is entirely in *timing*, and the basis return-period curves (Figure 6) make it precise.

Figure 12 gives the schematic intuition before the empirical curves: a bounded, AAL-neutral payout covers the body of the loss distribution but cannot follow the deep tail, so the insurer holds the bounded middle while the insured retains the extreme, and—reading by severity—the insured moves from a short-horizon surplus to a long-horizon shortfall.

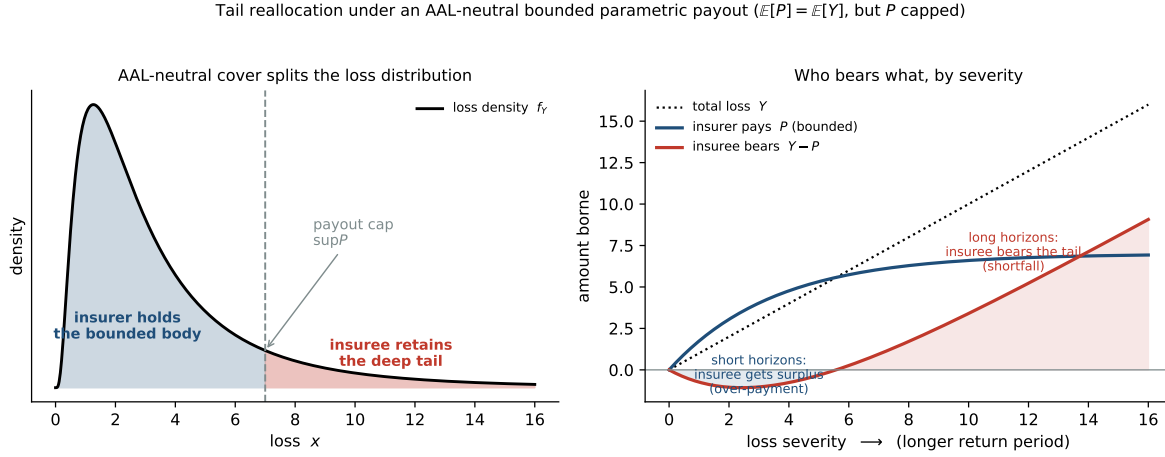


Figure 12: How an AAL-neutral bounded parametric payout reallocates the loss between insurer and insured (schematic). Left: the payout caps at $\sup P$, so the insurer absorbs the body of the loss density while the insured retains the deep tail beyond the cap; the means are equalised but the tail is not. Right: as severity (return period) grows, the parametric payout P saturates at its cap while the indemnity loss Y keeps rising, so the insured's retained loss $Y - P$ starts *negative* (a short-horizon surplus from over-payment) and then crosses zero and climbs (a long-horizon shortfall). The crossover is the horizon at which the advantage flips.

At short and moderate return periods the over-payment level turns positive before the shortfall level does. For the continuous design at R1, $\beta = 0.95$, the over-payment return level is already material at a 25-year horizon ($b_{25}^- \approx 13$) while the shortfall is still building, becoming material only at longer horizons. The mechanism is structural: over-payment occurs in the *common* states where the index trips and pays a bounded amount but the realised loss is small, whereas shortfall requires the *rarer* states in which exposure-driven severity outruns the capped payout. In the horizon band a policyholder actually experiences over a normal programme—the lower tens of years—the parametric contract therefore tends to pay them *more* than their realised loss. The insured benefits in the short run.

Beyond a crossover horizon the ordering reverses and shortfall dominates. Across the three regions the continuous-design crossover sits in the range $T \approx 60$ –90 years (R1 ≈ 61 , R2 ≈ 89 , R3 ≈ 91); past it, the shortfall return level exceeds the over-payment level and keeps climbing, reaching $b_{200}^+ \approx 70$ at R1 against an over-payment level of ≈ 38 . The binary design crosses earlier and far more sharply, because its over-payment is frozen at the fixed payout while its shortfall continues to grow: a digital payout simply cannot scale with a once-in-two-century exposure shock. In the deep tail it is the *insured* who is exposed—the bounded payout falls progressively further behind the unbounded indemnity loss, so the rare catastrophic year is borne by the policyholder, not the issuer. The crossover return period—several decades for the continuous design here, and materially shorter for the binary design—is the natural single number to report as the horizon at which the advantage flips.

9.2 Why this grows with catastrophe intensity

This temporal split is the key to a motivation for NatPar that is often asserted but rarely made precise. As catastrophe risk has intensified—larger exposures from economic growth, and more frequent and more severe hazard states from climate change—the right tail of $Y = A D$ has fattened: growth thickens the tail of exposure A , and climate change shifts and thickens the tail of damage D . A bounded parametric payout does not follow that tail. Holding the contract AAL-neutral, the consequence is not a change in the average transfer but an *enlargement of the deep-tail region that lies past the crossover horizon*—precisely the region the insurer sheds and the insuree absorbs. The heavier the catastrophe tail becomes, the more severe the post-crossover shortfall, and so the more the bounded structure relocates toward the insuree. This gives a clean, model-based statement of the popular intuition: *as catastrophe risk grows, an AAL-neutral parametric contract increasingly hands the insurer the manageable middle of the loss distribution while leaving the increasingly heavy tail with the insuree.*

It is important, however, to separate three distinct effects that the bounded structure delivers simultaneously, because only one of them is a transfer to the insuree, and conflating them overstates the case.

1. **Tail transfer (a cost to the insuree).** Deep-tail severity past the crossover horizon moves from insurer to policyholder, as above. This grows with catastrophe intensity and is the channel that raises suitability and conduct concerns.
2. **Capital and uncertainty relief (not a transfer).** Replacing exposure-driven indemnity loss with an index-driven bounded payout removes exposure uncertainty and development risk from the insurer’s liability, lowering VaR/TVaR per unit of expected loss and shortening the reserving tail. This relief comes from boundedness, verifiability, and the absence of a loss-adjustment tail; it would exist even if basis risk were zero, and it is the channel that makes the product acceptable to capital providers and supervisors.
3. **Liquidity (a benefit to the insuree).** Fast, auditable, index-verified settlement delivers timely cash in exactly the states where liquidity constraints bind—a genuine benefit to the policyholder that an indemnity contract, with slow loss adjustment, does not provide.

A complete account of NatPar’s rise must credit all three. The capital and liquidity channels explain why the product clears at all; the tail-transfer channel explains why its popularity tracks catastrophe intensity and why it warrants governance attention. Read this way, NatPar’s growth and the widening climate protection gap are two faces of the same phenomenon: when the loss tail grows faster than capacity and model certainty can follow, the market clears by bounding what is contractual—retreating to the part of the distribution that remains insurable under binding capital and uncertainty constraints, and leaving the rest shifted or uninsured.

9.3 Diversification and the relocation of the tail

The crossover horizon of Section 9 was computed risk-by-risk. A portfolio adds a second force—diversification—that interacts with the relocation in a way that is itself informative for both pricing and supervision. We isolate it with a controlled experiment. Exposures are taken *independent across regions* (the dollar value at risk in one region says nothing about another), while the hazard carries the *only* cross-region dependence through a one-factor model: each regional index is $Z_r = \sqrt{\rho} F + \sqrt{1 - \rho} \varepsilon_r$ with a common climate factor F and idiosyncratic ε_r , so ρ tunes the probability

that cold extremes strike several regions in the same season. Each design is re-priced at the AAL-neutral optimal trigger within each region, and we read the *aggregate* basis $S^B = \sum_r (Y_r - P_r)$. Table 10 and Figure 13 report the result.

Diversification smooths the body but not the tail, pulling the crossover earlier. Under independent hazard ($\rho = 0$) the aggregate-basis crossover for the continuous design falls to roughly 30–40 years, shorter than the ≈ 60 –90-year standalone crossover of a single region. The reason is asymmetric. The frequent, mild over-payments are largely idiosyncratic across regions and partly net out when summed, so the *body* of the aggregate basis is compressed by pooling. The severe shortfalls, by contrast, sit in the heavy *AD* tail and do not cancel: a bad year in one region still contributes its full unspanned loss to the sum. Diversification therefore thins the over-payment side faster than the shortfall side, and the shortfall overtakes over-payment sooner in aggregate than it does for a lone risk. For the insuree this is a double-edged outcome: pooling stabilises the routine over-payment surplus they enjoyed at short horizons, but it also brings forward the horizon at which they begin to bear net shortfall.

Correlated hazard erodes the diversification and hands the deep tail back to the insurer. As ρ rises the common climate factor synchronises regional cold extremes, so the regions stop diversifying in precisely the states that matter. The right panel of Figure 13 tracks the 200-year dominance margin $b_{200}^+ - b_{200}^-$: it is firmly positive at low ρ (the insuree bears the deep tail, as in the single-risk analysis) but falls toward zero and turns *negative* as $\rho \rightarrow 1$. Under near-comonotonic hazard the bounded payouts of all regions trigger together, stacking into a large aggregate payout in the common catastrophic year while the realised aggregate loss, though large, is matched or exceeded by that stacked payout; the insurer then *over-pays* in the tail and reabsorbs the very cost that independence had relocated to the insuree. Table 10 shows the aggregate basis variance, which is essentially the sum of standalone variances at $\rho = 0$ (the diversification ratio is ≈ 1.00 , confirming near-independence in the base calibration), rising above it as $\rho \rightarrow 1$ (ratio ≈ 1.08 continuous, ≈ 1.26 binary): correlation destroys the pooling benefit and concentrates risk back into the aggregate tail.

Implications. Three points follow. First, the tail relocation of Section 9 is not a fixed property of a contract but depends on the portfolio it sits in and on the dependence among hazards: the same AAL-neutral design can leave the deep tail with the insuree on a diversified, weakly-correlated book and with the insurer on a concentrated, highly-correlated one. Second, the binary design is the more dependence-sensitive of the two—its dominance margin swings furthest as ρ moves—because its fixed payouts stack most mechanically under comonotonicity; digitisation that looks capital-light on an independent book can become capital-heavy on a correlated one. Third, and most relevant under climate change, the spatial correlation of extremes is itself increasing: large-scale climate drivers raise ρ over time, which by this mechanism partially *returns* deep-tail cost to insurers even as growing exposure fattens the tail in the first place. The two climate forces therefore push in opposite directions on who ultimately bears the tail, and the occurrence-basis object M^B remains the right supervisory lens because it captures the worst single-region shortfall regardless of how the aggregate nets out.

Table 10: Diversification and the aggregate basis (continuous and binary, $\beta = 0.95$, independent exposures). Crossover return period; 200- and 50-year shortfall (b^+) and overpayment (b^-) levels; and the diversification ratio $\text{Var}(S^B)/\sum_r \text{Var}(B_r)$, as the cross-region hazard correlation ρ varies.

ρ	Design	crossover T	b_{50}^+	b_{50}^-	b_{200}^+	b_{200}^-	div. ratio
0.0	c	32	40.56	34.01	84.92	48.80	1.00
0.0	b	10	42.23	32.37	88.58	34.77	1.00
0.3	c	34	39.03	33.69	83.77	49.71	1.00
0.3	b	38	40.40	32.15	86.05	45.82	0.99
0.6	c	37	37.63	33.57	83.21	53.36	1.00
0.6	b	12	39.22	34.19	85.16	62.21	0.99
1.0	c	105	34.25	42.32	84.53	69.88	1.05
1.0	b	15	35.22	44.85	84.79	76.53	1.05

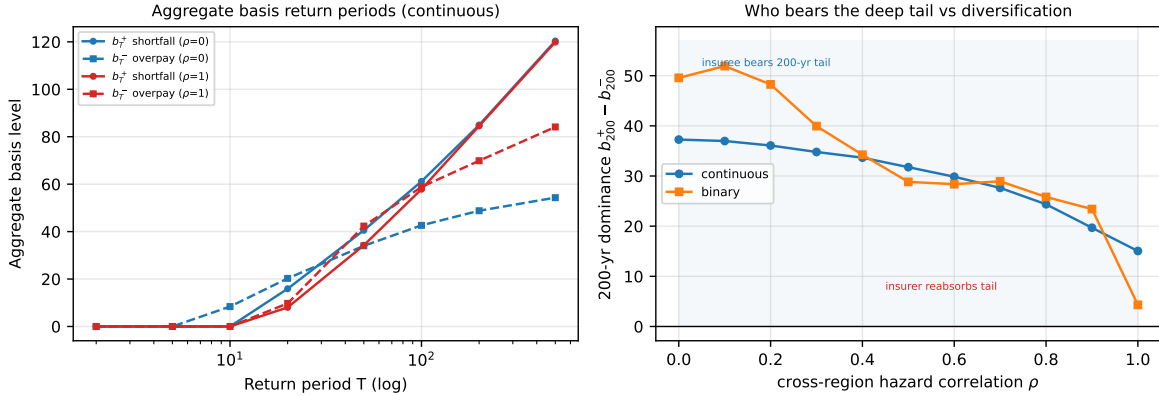


Figure 13: Diversification and the relocation of the tail (independent exposures, correlated hazard). Left: aggregate-basis return-period curves for independent ($\rho = 0$, blue) versus comonotonic ($\rho = 1$, red) hazard; under independence the shortfall side (solid) dominates the tail, while under comonotonicity the overpayment side (dashed) catches up and overtakes it. Right: the 200-year dominance margin $b_{200}^+ - b_{200}^-$ as a function of ρ ; positive values mean the insured bears the deep tail, negative values mean the insurer reabsorbs it. The binary design is the more dependence-sensitive.

9.4 Tail dependence, not correlation, governs the deep-tail relocation

The diversification experiment of Section 9.3 used a one-factor Gaussian model, in which cross-region dependence is summarised by a single linear correlation ρ . For catastrophe perils this understates the danger in exactly the region that matters. Correlation is a whole-distribution, linear measure, and the Gaussian copula has *zero tail dependence*: the conditional probability that two regions hit their extremes together vanishes as one moves further into the tail, however high ρ is set. Yet the defining feature of climate perils is the opposite—a continental cold outbreak drives regions to their cold extremes *jointly*. The relevant question is therefore not how correlated the regions are on average, but how strongly they fail together in the tail.

We isolate this by holding the linear correlation fixed and varying only the coefficient of lower tail dependence λ_L , replacing the Gaussian copula on the regional hazards with a t -copula whose

degrees of freedom ν control λ_L (smaller ν , heavier tail dependence; the Gaussian is the $\nu \rightarrow \infty$ limit with $\lambda_L = 0$). As noted in Remark 3, this changes only the copula: each regional temperature stays exactly Gaussian, so tail dependence is introduced without giving any single hazard a heavy marginal tail. Exposures remain independent across regions, and frost is mapped as a lower extreme so that the copula’s joint tail corresponds to joint cold. Table 11 and Figure 14 report the effect.

Correlation can be blind to the joint catastrophe. The left panel of Figure 14 shows draws of the regional hazard quantiles under a Gaussian copula and under a t -copula with $\nu = 2$, calibrated to the *same* linear correlation $\rho \approx 0.3$. The two are nearly indistinguishable through the body, but the t -copula packs visibly more mass into the joint lower-left corner—both regions simultaneously in their cold extremes. Table 11 quantifies this: as λ_L rises from 0 (Gaussian) to ≈ 0.29 ($\nu = 2$), the linear correlation barely moves (it stays near 0.29), yet the probability that all three regions fall below their fifth hazard percentile in the same season roughly quadruples, from about 0.0017 to 0.0073. A correlation-only diagnostic would declare these portfolios equivalent; they are not.

Heavier tail dependence returns the deep tail to the insurer. Because the joint catastrophe becomes more frequent, the bounded parametric payouts of the several regions increasingly trigger *together* in the same severe year, stacking into a large aggregate payout precisely when independence would have kept the portfolio loss moderate. The right panel of Figure 14 tracks the 200-year dominance margin $b_{200}^+ - b_{200}^-$ against λ_L at fixed correlation: it declines monotonically for both designs as tail dependence strengthens—from about 23 to 13 for the continuous design and from about 35 to 17 for the binary design over the range studied. The deep tail is being handed back from the insuree toward the insurer purely through tail dependence, with correlation held constant. This is the same reabsorption mechanism that the comonotonic limit produced in Section 9.3, but here it is driven by the genuinely relevant quantity and it operates well before correlation reaches its extreme: even a moderately heavy-tailed coupling materially shifts who carries the once-in-two-century year.

Implications. First, the relocation results of this paper should be read against tail dependence, not correlation: two books with identical correlation and identical AAL-neutral contracts can allocate the deep tail very differently depending on how their hazards behave in the joint extreme, and only the tail-dependence view distinguishes them. Second, the binary design is again the more dependence-sensitive—its margin sits higher but falls faster—because its fixed payouts stack most mechanically when regions trigger together. Third, the climate implication sharpens the discussion of Section 9: climate change is widely expected to increase not just the marginal severity of extremes but their *spatial co-occurrence*, i.e. λ_L itself. By the mechanism here, rising tail dependence pushes deep-tail cost back toward insurers even as fatter marginal tails push it toward insurees, so the net incidence of catastrophe risk under NatPar is the resultant of two opposing climate forces, neither of which is visible to a correlation-based portfolio model. For supervision this reinforces the case for the occurrence basis M^B and for stress tests specified in terms of joint-extreme (tail-dependence) scenarios rather than correlation matrices.

Table 11: Tail dependence versus correlation (continuous and binary, $\beta = 0.95$, independent exposures, fixed target correlation $\rho = 0.3$). As the lower tail-dependence coefficient λ_L increases (via the t -copula degrees of freedom ν), the realised correlation is essentially unchanged while the joint cold-extreme probability $\mathbb{P}(\text{all below } 5\%)$ rises and the 200-year dominance margin $b_{200}^+ - b_{200}^-$ falls.

ν	Design	λ_L	$\widehat{\text{Corr}}$	$\mathbb{P}(\text{joint } 5\%)$	b_{200}^+	b_{200}^-	margin
Gauss	c	0.00	0.30	0.0017	88.80	50.04	38.76
Gauss	b	0.00	0.30	0.0017	90.22	49.70	40.52
8	c	0.06	0.29	0.0030	84.80	50.03	34.77
8	b	0.06	0.29	0.0030	88.22	52.43	35.79
4	c	0.16	0.29	0.0044	83.38	51.16	32.21
4	b	0.16	0.29	0.0044	86.91	56.88	30.03
2	c	0.29	0.28	0.0073	82.47	55.89	26.58
2	b	0.29	0.28	0.0073	85.67	60.43	25.24

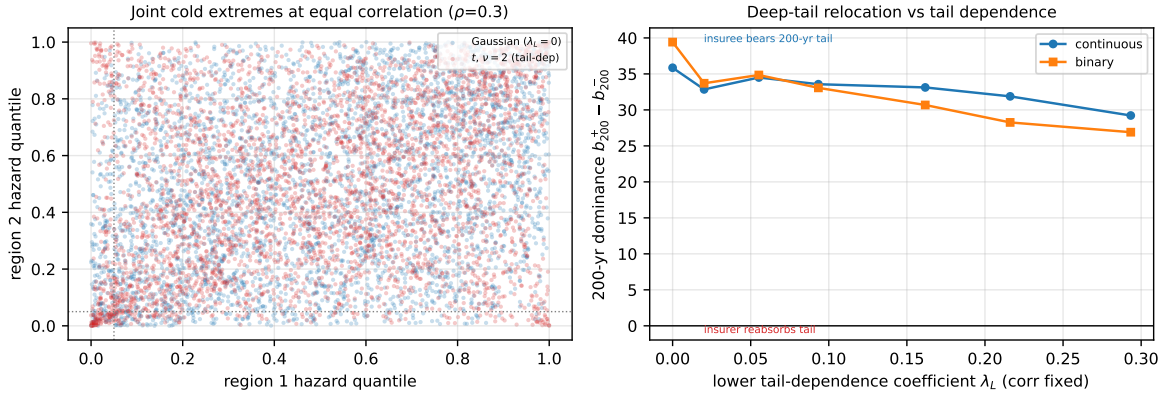


Figure 14: Tail dependence, not correlation, governs the deep-tail relocation. Left: regional hazard-quantile draws under a Gaussian copula ($\lambda_L = 0$) and a t -copula with $\nu = 2$, both at the same linear correlation $\rho \approx 0.3$; the t -copula concentrates mass in the joint lower-left corner (joint cold extremes) that the Gaussian misses. Right: the 200-year dominance margin $b_{200}^+ - b_{200}^-$ as a function of the lower tail-dependence coefficient λ_L at fixed correlation; heavier tail dependence monotonically shifts the deep tail from insuree toward insurer, with the binary design the more sensitive.

9.5 A testable hypothesis, not yet an empirical claim

We frame the above as a hypothesis that the NatPar reporting framework is built to express and test, rather than as an established empirical result. The frost case study is a stationary, single-season illustration; demonstrating the relocation empirically would require a contract held fixed while exposure and hazard distributions are made non-stationary over time, so that the crossover horizon can be tracked as the tail fattens. The apparatus needed for that test—AAL-neutral calibration to hold the mean fixed, basis return-period curves to expose the timing, and the crossover horizon as a summary statistic—is exactly what the framework provides. We regard quantifying the relocation under explicit climate and exposure-growth scenarios as the natural next application, and note

that the occurrence-basis object M^B of Section 4 is the appropriate lens for supervisors, since it captures the worst single-region shortfall in a year rather than a portfolio average that may net out.

10 Extensions and discussion

The frost example shows, in a controlled setting, how NatPar emerges from an existing NatCat model and how it changes the distribution of losses and capital-relevant metrics. We sketch several extensions and discuss broader implications and limitations.

10.1 Multi-trigger and multi-peril NatPar programmes

Real-world parametric programmes often involve multiple indices, multiple perils and layered structures. Within a single peril it is natural to consider multi-trigger structures using several aspects of the hazard, $P_{e,m} = I_m(Z_e^{(1)}, \dots, Z_e^{(J)})$ with $J > 1$ indices per event. The counting argument of Section 5 gives a clean design rule here: each additional contractible index is also an additional free parameter, and so buys one more exactly-satisfiable moment condition (e.g. AAL-neutrality *and* tail-neutrality). Multi-trigger designs are attractive when they capture qualitatively different damage channels difficult to encode in a single index, and become problematic when the payout surface is opaque or when small parameter changes produce large, unintuitive payout changes. If P indexes perils and R indexes regions, a general NatPar portfolio is $S^{\text{Par}} = \sum_{p \in P} \sum_{r \in R} S_{p,r}^{\text{Par}}$, driven by the same event-level hazard simulations that drive the NatCat model.

Complexity vs transparency. NatPar provides a counterweight to the temptation to exploit the full flexibility of $I(X)$: because the same NatCat models estimate the incremental reduction in basis risk from each added index, one can compare the benefit (in basis-risk metrics) against the cost (transparency, operational complexity, regulatory acceptability). In many cases a small number of well-chosen simple triggers dominates more elaborate designs.

10.2 NatPar and solvency/climate-risk regulation

Regulatory capital regimes typically impose risk measures such as VaR or TVaR at high confidence over one-year horizons. As shown in the frost example, NatPar portfolios often have shorter tails and lower TVaR for a given AAL than their indemnity counterparts, due to bounded, index-linked liabilities and the absence of a long development tail; under tail-sensitive capital regimes they can therefore be more capital-efficient per unit of expected loss. This raises both opportunities and questions: parametric structures may allow insurers to maintain coverage in climate-exposed lines while respecting capital requirements, but they introduce basis risk that must be understood and, in some cases, explicitly reported or capitalised. Climate-risk stress testing typically alters the hazard module and re-runs the model; in a NatPar setting the same altered hazards feed directly into index distributions and parametric payouts, and because NatPar is more tightly linked to hazard and less to exposure it can give sharper insight into how balance sheets respond to physical climate shifts. The NatPar framework also offers a way to present parametric programmes in a language familiar to regulators—as variants of existing NatCat models with clear, quantifiable basis-risk profiles—and suggests that guidance may wish to distinguish simple hazard-linked structures from highly complex multi-trigger designs.

10.3 Limitations and practical challenges

Data and model limitations. NatPar relies on the same underlying hazard, exposure and vulnerability models as NatCat. If these are poor, biased or unstable, the resulting basis-risk analysis and parametric designs will also be unreliable. In some markets hazard data are sparse, exposure data incomplete, and loss histories short; in others, non-stationarity is so pronounced that historical patterns may be a poor guide to the future. NatPar does not remove these problems; it re-allocates them. In particular, the empirical calibration here uses a Gaussian fit to 65 seasonal observations per region and a stylised lognormal exposure; richer marginals and dependence (e.g. copulas, or $A_r | T_r$ regression) can be substituted without changing the NatCat–NatPar logic.

Governance of indices and triggers. Parametric payouts are only as good as the indices they rely on. This raises questions of governance: who controls the index, how revisions or errors are handled, how disputes are resolved, and what happens when measurement systems change. Robust contractual and institutional arrangements are needed to keep indices trustworthy over the lifetime of a programme.

Market acceptance and client understanding. Even simple parametric structures can be unfamiliar to policyholders and intermediaries used to indemnity insurance. Basis risk in particular can be hard to explain: clients may perceive a contract as unfair if it fails to pay in a year when they experience losses, even if the design is actuarially sound. Education, clear documentation, and careful trigger choice are essential for market acceptance.

Operational and legal considerations. Implementing NatPar programmes at scale requires robust operational processes: index calculation and verification, payout automation, integration with underwriting and claims systems, and coordination with reinsurance and capital market transactions. Legal frameworks for parametric products may be less well-developed than for traditional insurance in some jurisdictions.

11 Conclusion

Starting from a simple NatCat model of seasonal frost damage in three simulated regions sharing one identical damage function, we defined an indemnity cover and constructed two NatPar alternatives whose payouts depend only on temperature: a continuous design $P = qD(T)$ and a binary tail-trigger design $P = q\mathbf{1}\{T < \tau\}$, calibrated by AAL-neutrality and priced by minimising a loss of mismatch. Because the designs share the same hazard and vulnerability base, differences in EP curves, VaR, and basis behaviour are attributable to contract structure rather than to inconsistent modelling inputs, and several conclusions emerge that are generic enough to set a standard for how NatPar programmes should be evaluated.

The first is that payout shape, not the parametric label, governs tail capital. “Parametric” is not automatically “capital-light”: continuous bounded schedules tend to cap liabilities and dampen exposure-driven tails, whereas digital schedules can concentrate losses into capital-relevant quantiles, so trigger probability and payout shape must be judged jointly against the chosen capital metric. The second is that basis risk is the operational interface between NatCat and NatPar rather than a nuisance term. Because NatPar replaces the exposure–vulnerability component of indemnity loss with a low-dimensional hazard-only payout, the residual $B := Y - P$ is a measurable object, and the exceedance-basis curves give it a tail view of shortfall and overpayment that

is directly suited to governance and model validation. The third concerns pricing: the trigger price is the solution of minimising the expected disutility of the two-sided basis, and what moves that price is the curvature of the penalty, not merely its weight. Asymmetric curvature on shortfall versus overpayment shifts the premium decisively along the shortfall/overpayment trade-off, while a change of weight under a linear penalty leaves it at the AAL-neutral value; fair pricing is therefore the symmetric-linear corner of the problem rather than a competing paradigm, and because the number of conditions a price can satisfy exactly is bounded by the contract's free parameters, over-determination is cured by adding a parameter rather than by fighting the algebra.

Beyond the single risk, the portfolio analysis shows that who bears the catastrophe tail is not a fixed property of a contract but depends on the book it sits in and on how its hazards behave jointly. Under an AAL-neutral design the mismatch is relocated across time rather than on average: the insuree tends to gain at short and moderate horizons, where bounded over-payments exceed realised losses, while the insurer sheds the deep tail past a crossover horizon of order a century. Diversification across weakly dependent regions pulls that crossover earlier, because the frequent over-payments net out while the severe shortfalls do not. The direction reverses, however, once the hazards exhibit tail dependence: when regions reach their cold extremes jointly the bounded payouts stack and the insurer reabsorbs the deep tail, an effect driven by tail dependence at fixed correlation and therefore invisible to a correlation-based portfolio model. The net incidence of catastrophe risk under NatPar is thus the resultant of two opposing climate forces—fatter marginal tails pushing the deep tail toward the insuree, rising spatial co-occurrence of extremes pushing it back toward the insurer—which is why we regard the occurrence basis and joint-extreme stress scenarios, rather than averages and correlation matrices, as the right supervisory lenses.

Taken together, these results support a disciplined reporting template that extends the familiar NatCat outputs to the parametric portfolio and pairs them with basis-risk exceedance diagnostics and the loss-minimisation price, making NatPar programmes comparable, auditable, and amenable to supervision while discouraging gratuitous complexity, since additional triggers and indices should be justified by measured basis-risk reduction net of the transparency and operational robustness they cost. Natural Parametric insurance, on this reading, is not a departure from NatCat practice but a reorganisation of it in response to climate-driven uninsurability, exposure uncertainty, and capital pressure—and making that reorganisation explicit gives theorists, practitioners, and supervisors a common language in which to design, analyse, and govern parametric programmes in a way that is both mathematically coherent and grounded in how catastrophe risk is actually managed.

A Proof of equation (14)

Fix $x \geq 0$ and condition on D . *Case 1 (no loss under deductible, $AD \leq l$):* then $Y = 0$ and $P - Y = qD'$, so overpayment exceedance is $qD' > x$, which depends only on hazard. *Case 2 (above deductible, $AD > l$):* then $Y = AD - l$ and $P - Y = qD' - (AD - l) = l + qD' - AD$, so $P - Y > x \Leftrightarrow A < (l - x + qD')/D$, together with $A > l/D$. Combining the two cases,

$$\begin{aligned} \text{BEP}^{c,-}(x) &= \mathbb{P}(P - Y > x) = \mathbb{E}[\mathbf{1}\{P - Y > x\} \mathbf{1}\{D > 0\}] \\ &= \mathbb{E}\left[F_A\left(\frac{l}{D}\right) \mathbf{1}\{D > 0, qD' > x\}\right] + \mathbb{E}\left[\left(F_A\left(\frac{l+qD'-x}{D}\right) - F_A\left(\frac{l}{D}\right)\right) \mathbf{1}\{D > 0, qD' > x\}\right] \\ &= \mathbb{E}\left[F_A\left(\frac{l+qD'-x}{D}\right) \mathbf{1}\{qD' > x\}\right] = \mathbb{E}\left[F_A\left(\frac{l-qd-x}{D} + q\right) \mathbf{1}\{D > x/q + d\}\right]. \end{aligned}$$

B Proof of equation (15)

$$\begin{aligned}
\text{BEP}^{b,+}(x) &= \mathbb{P}(Y > x, D \leq d) + \mathbb{P}(Y > x + q, D > d) \\
&= \mathbb{P}(AD > x + l, D \leq d) + \mathbb{P}(AD > x + q + l, D > d) \\
&= \mathbb{E}\left[\overline{F}_A\left(\frac{x+l}{D}\right) \mathbf{1}\{D \leq d\}\right] + \mathbb{E}\left[\overline{F}_A\left(\frac{x+q+l}{D}\right) \mathbf{1}\{D > d\}\right].
\end{aligned}$$

C Proof of equation (16)

$$\begin{aligned}
\text{BEP}^{b,-}(x) &= \mathbb{P}(-B > x) = \mathbb{P}(q > x, D > d, Y = 0) + \mathbb{P}(q - Y > x, D > d, Y > 0) \\
&= \mathbb{P}\left(q > x, D > d, A \leq \frac{l}{D}\right) + \mathbb{P}\left(q > x, \frac{q+l-x}{D} > A > \frac{l}{D}, D > d\right) \\
&= \begin{cases} \mathbb{E}\left[F_A\left(\frac{q+l-x}{D}\right) \mathbf{1}\{D > d\}\right], & x < q, \\ 0, & x \geq q. \end{cases}
\end{aligned}$$

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